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Streszczenie

W niniejszym artykule analizujemy dwie kluczowe kwestie dotyczące projektowania optymalnego systemu podatkowego, wykorzystując polskie reformy podatkowe z 2009 r. Po pierwsze, szacujemy stopień zastąpienia dochodu z zatrudnienia - dochodem z własnej działalności gospodarczej, na podstawie ekstensywnej zmienności. W szczególności określamy wpływ zmian w różnicach stawek opodatkowania zatrudnienia a opodatkowania własnej działalności na skłonność podatników do deklarowania dochodów z zatrudnienia lub z działalności gospodarczej. Po drugie, przyczyniamy się do wzbogacenia literatury dotyczącej elastyczności dochodów względem krańcowej stopy podatkowej, dostarczając szacunków, które są niezależne od zmian dynamiki dochodów z roku na rok. Czynimy to poprzez wykorzystanie zmian krańcowych stawek podatkowych w następstwie reform z 2009 r., które występują niezależnie od położenia podatnika w rozkładzie dochodu.

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Podstawowe szacunki elastyczności intensywnej zmienności względem krańcowej stawki podatku wynoszą 0,23 dla zatrudnionych oraz 0,66 dla osób prowadzących działalność gospodarczą. Oszacowania umożliwiają dekompozycję zmian deklarowanych dochodów w wyniku reformy podatkowej z 2009 r. na kategorie intensywne i ekstensywne, przy czym wkład ekstensywnej kategorii wynosi około 7 procent całości zmian.

Abstract

In this paper we examine two crucial questions regarding the design of the optimal tax system, exploiting the 2009 Polish tax reforms. Firstly, we estimate the degree of substitution between the employment and self-employment tax bases on the extensive margin. In particular, we quantify the impact of changes in the differential in rates of taxation between the two tax bases on the propensity of taxpayers to declare any positive level of employment or self-employment income. Secondly, we contribute to the literature on elasticities of taxable income by providing estimates which are robust to changes in year-to-year income dynamics. We do this by exploiting variation in marginal tax rates around the 2009 reforms which occurs independently of an individual's position in the income distribution as a result of joint reporting with a spouse. The baseline estimates of the intensive-margin elasticities are 0.23 for the employed and 0.66 for the self-employed. The estimates jointly make possible a decomposition of responses to the tax reform of declared income into the intensive and extensive-margins, with the contribution of the extensive margin found to be around 7 percent of the total.

1 Introduction

Under certain conditions (Feldstein, 1995), the elasticity of taxable income (ETI) is a sufficient statistic to measure the efficiency of taxation, making it a central parameter for optimal tax policy.[1] However, although the empirical literature on ETIs is extensive, there is still uncertainty over the reliability and relevance of existing elasticity estimates. This paper addresses two sets of challenges which cast doubt over the robustness of existing empirical studies of such elasticities: the presence of fiscal externalities and the lack of stability in income dynamics over time.

The first set of challenges concerns the degree to which changes in reported income at the time of tax reforms reflect genuine taxable income responses, or are merely a result of tax-base shifting. In the latter case, income is either reclassified from other tax bases or taxpayers substitute between different types of taxed economic activity. Such responses to tax reforms have been termed *fiscal externalities* (Saez et al., 2012, see e.g.). If the ETI is driven partly by a shifting response, it ceases to be a sufficient statistic in optimal tax formulae. From the perspective of optimal tax theory, different elasticities on different types of income may imply a wedge in marginal tax rates between different types of income. For instance, previous studies, such as Kopczuk (2015), suggest that business/self-employment income has a higher ETI than employment income, suggesting a *prima facie* case for taxing business/self-employment income at a lower rate than income from employment. However, it is unclear to what degree such differential taxation would create fiscal externalities. If a differential in tax rates induces substitution between tax bases, ETIs need to be complemented by estimates of cross-elasticities. These measure how the level of reported income of one category changes as a result of changes in the tax rates in another category. For instance, Gordon and Slemrod (2000) argue that the response to the Reagan 1986 tax cuts in the personal income tax base may have been driven mainly by taxpayers shifting from the corporate tax base.

A second set of challenges is methodological, and surrounds the identifi-

[1] Saez, Slemrod and Giertz (2012) provide a comprehensive survey.

cation of ETI parameters. The dominant empirical approach has been to use tax reforms which create exogenous variation in marginal tax rates (MTRs) for some sub-groups of taxpayers while leaving them unchanged for others. The change in reported taxable income in an affected group (analogous to a 'treatment' group in a randomised controlled trial (RCT)) is compared to changes in the unaffected group (analogous to a 'control' group), and the difference is attributed to a behavioural response to tax reforms.[2] It is unclear whether existing studies adequately control for the changes in taxable income which would have occurred anyway, even in the absence of tax reforms. Since tax reforms are usually concentrated in certain sub-sections of the income distribution, researchers have tended to assume that the pattern of income growth across the income distribution is stable over time, absent reforms. This, however, may not hold in practice. For instance, business cycle factors affect some parts of the income distribution more than others, income inequality trends may also change over time and, finally, people at different points in the distribution and different types of income may have different elasticities (Saez et al., 2012, discuss this in their review chapter).

This paper seeks to address both of these concerns by taking advantage of the details of a tax reform episode which occurred in Poland in 2009. Since the Polish tax reform altered the relative difference in the tax burdens between an employed individual and a self-employed individual with the same income, it allows the estimation of fiscal externalities between these two tax bases. The fiscal externalities occur on the extensive margin, whereby individuals switch entirely between self-employment and employment activity.[3] The reform also allows the estimation of traditional intensive-margin ETIs

[2]Classic papers in this spirit include Feldstein (1995), Gruber and Saez (2002) and Kopczuk (2005). The labels 'treatment' and 'control' groups usually used in the literature do not have the same meaning as in the RCT literature, since treatment is not randomly assigned.

[3]An alternative would be to allow there to be fiscal externalities on both the intensive and extensive margins, which we do in a draft version of this paper (available on request). In this scenario, individuals with both types of income adjust the relative amount of income declared in each tax base. Since the vast majority of taxpayers report either one or the other type of income, however, we focus on the extensive-margin in the main text.

for the two main types of income under consideration alongside the fiscal externalities. Looking at the intensive and extensive-margins to tax reform together is complementary since both margins of response are likely to matter, analyzing them together in the context of one reform episode may give us an idea of the relative importance of the responses from a policymaker's perspective.

On the intensive margin, the reform episode permits an identification strategy which does not require the assumption of stable income dynamics over time across the income distribution. For every individual affected by the reform, we are able to find an individual who was not affected by the reform with the same level of income in other words, the tax reform caused variation in MTRs independently of an individual's position in the income distribution. The identification stems from the option to file jointly with spouses, and the fact that taxpayers experienced differential changes in MTRs due to *different levels of spousal income*. Joint filing with a lower-income spouse may allow an individual to enter a lower income tax bracket than under single filing, while an individual with the same income but a higher-income spouse may be forced to remain in a higher tax bracket. It is the latter taxpayers who experienced a large cut in MTRs as a result of Poland's 2009 tax reforms, while the former did not. In this manner, we obtain a treatment and a control group which is independent of the position in the taxpayer's own position in the income distribution. Because of this, it also becomes possible to study the degree to which the elasticity changes across the income distribution. For estimating extensive-margin elasticities, we rely both on an analogous estimation strategy, where individuals with spouses provide a control group to individuals transitioning to and from the linear business tax base, as well as an alternative strategy based on transitions around income brackets.

The main assumption for identifying intensive-margin elasticities used here is that, absent reforms, changes in the taxable income declared by a taxpayer are independent of his or her spouse's income level, once we condition for the taxpayer's own base-year income. On the face of it, this would appear to be a strong assumption. However, due to the availability of several years where no reforms occurred, we can verify its validity by conducting placebo tests something which is unfortunately rarely done in the literature (Kopczuk (2015) is a notable exception). The paper finds strong evidence that individuals with low-income spouses do indeed constitute a convincing control group for individuals with high-income spouses and the same base-year income. Moreover, a lack of stability in income dynamics from year to year further suggest that traditional approaches to ETI estimation, such as the Gruber-Saez approach, would have yielded biased estimates in the current dataset of Polish taxpayers. Likewise, extensive-margin estimates are validated by placebo estimates in non-reform years.

Interestingly, we find stable own-elasticities of employment income across the income distribution in the 0.20-0.3 range. Own-elasticities of business income are higher, in the 0.5-0.7 range, although it is more difficult to estimate how stable these are across the income distribution because of small sample size. There are also non-trivial elasticities occurring at the extensive margin: a 10,000zł ($2,672 as of May 2017) change in the relative tax burden between the two tax bases increases the probability of a taxpayer filing linear business income, as opposed to progressive employment income, by 3.75 percentage points. We demonstrate how these intensive-margin elasticity estimates and extensive-margin responses matter together for the deadweight loss of tax reforms. We also use our estimates to calculate the likely contributions of the intensive and extensive margins to changes in deadweight losses at the time of the 2009 reforms in Poland.

The paper links to several strands in the public finance literature. Most closely related is Kopczuk (2015), who examines business income elasticities around the introduction of the linear tax option for business income in 2004 in Poland using a methodology also based on joint filing with a spouse. This study is complementary to his - a similar identification strategy is applied to a sample of business owners for intensive-margin elasticities. However, this paper does not examine the extensive-margin implications of differential taxation of tax bases. Kleven and Schultz (2014) examine cross-elasticities between business and employment income using Danish reforms with a methodology based on bunching. However, they focus purely on intensive-margin responses. From a theoretical perspective, Kleven et al. (2009) is a seminal contribution focusing on the joint taxation of a household involving a primary taxpayer and the spouse. The decision whether to work or not of a taxpayer's spouse in this paper shares many similarities with the choice to switch tax bases here. Thus, Scheuer (2014) applies an analogous framework to Kleven et al. (2009) to examine optimal non-linear taxation in environment with an extensive-margin occupational choice between employment and business. This approach to combining intensive and extensive-margin decisions will be followed here as well.

The structure of the remaining chapter is as follows. Section 2 gives background information on the Polish tax system and the 2009 tax reform. Section 3 develops a model of tax reporting in an environment with multiple tax bases and demonstrates how fiscal externalities affect the formulae for the of tax reforms, which also describe tax revenue changes due to behavioural responses. It also illustrates how extensive-margin switching parameters enter fully optimal non-linear tax formulae. Section 4 outlines the dataset used. Sections 5 and 6 present the empirical strategy results on the intensive and extensive margin, respectively. Section 7 applies these results to estimate the revenue effects and of the Polish tax reform. The chapter concludes in Section 8.

2 Background: The 2009 Polish Tax Reform

The basic Polish personal income tax system has a progressive structure. For most individuals, income from standard employment contracts and non-standard contracts, such as commissions, are aggregated with business income and subjected to a progressive personal income tax schedule. Before 2009, the schedule had three tax bands with increasing marginal tax rates: 19%, 30% and 40%, and a small tax-free allowance at the bottom. Unlike many other tax systems, however, as of 2004 the Polish tax code gave taxpayers with self-employment income a choice regarding how it is taxed. Individuals with such income are given the option of having self-employment

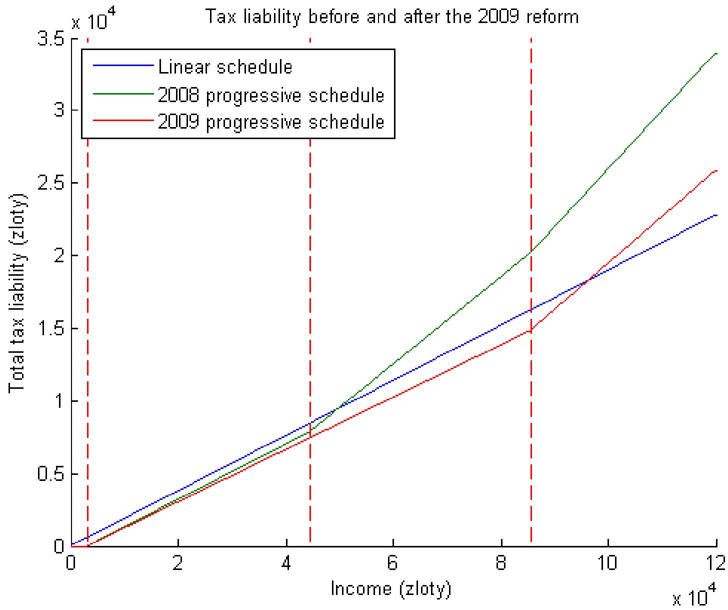

Figure 1: Tax liability pre and post-2009 reform. Both income and tax liability are in units of the Polish currency (zł). The green line represents the 2008 (pre-reform) progressive schedule, the red line represents the after-2009 (post-reform) progressive schedule, and the blue line represents the linear schedule for business taxation which remained unchanged throughout the reform. The vertical lines represent the position of the tax-bracket thresholds.

income taxed separately according to a linear 19% tax schedule.[4] The linear schedule deprives them of several tax advantages, including the tax-free allowance, the ability to claim deductions and the ability to share tax liability with a spouse. Income from assets is taxed separately according to a linear tax of 19%.

If a taxpayer is married, they have the option of reducing their tax liability through joint filing, a feature the Polish tax system shares with many other

[4]Self-employment income is here defined as income from unincorporated businesses which act as pass-through entities (I otherwise refer to it as "business income"). Income from incorporated businesses is taxed separately, once under the Corporation Income Tax (CIT), which is 19%, and subsequently again under a 19% personal tax which applies to capital gains.

tax systems, such as the US system. Income is summed over the taxpayer and the spouse: if the sum falls below twice a bracket threshold, a lower marginal rate is applied to the entire income, even if one of the taxpayers individually would have crossed the bracket and had been subject to a higher marginal rate. The averaging of incomes always creates an incentive to file jointly if the taxpayer would have otherwise been in a higher bracket than the spouse, or vice versa. Single parents can similarly share tax liability with their child, who acts as if they were a non-earning spouse.

The 2009 tax reform cut the the marginal tax rate in the bottom bracket by 1% and extended this tax bracket to cover what was previously the middle tax bracket, resulting in a 12% fall in the marginal tax rate for what was previously the middle tax bracket. The top rate was also reduced from from 40% to 32%. The reform therefore 'flattened' the tax schedule, resulting in a schedule with just two tax bands of 18% and 32%. The changes in marginal tax rates and thresholds are summarized in Table 1, while the change in the schedule relating total tax liability in terms of net income for a household as a result of the reform is illustrated in Figure 1. Crucially, since the optional linear tax schedule for self-employment income remained unchanged, the reform also had the effect of reducing the differential in tax rates between the progressive and linear schedules, especially for individuals who would have previously fallen in the second and third tax brackets in the progressive schedule.

Since the identification strategy used in this paper relies on joint filing by spouses, it is important to note that following the reform, the combinations of income for which there were positive gains from joint filing shrunk. For instance, if the original filer was in the middle tax bracket with a spouse in the lower tax bracket, following the reform there was no longer a gain from filing jointly (although there was no financial loss to doing so either). We do indeed see a fall in the proportion of individuals in the second bracket reporting with a spouse from 65.3% to 61.4%, while we do not see a similar fall for those in the third bracket. We largely abstract from this issue, and it will be seen that the population of those who continued to file jointly throughout the period continues to serve as a source of suitable treatment and control groups.

In the periods 2004-2006 and 2008-2012, the tax brackets remained frozen in nominal terms, thus implying many individuals would have experienced a transition into higher tax brackets due to inflation and secular income growth (so-called 'bracket-creep'). No significant changes to the tax base occurred in our sample period, and available deductions remained fairly constant. This implies that the definition of taxable income remained fairly stable, and thus merits focus on taxable income as the variable of interest (other studies have tended to focus on broad income, which has often been seen as the measure less sensitive to changes in definitions of the tax base). Significant real-GDP growth occurred in 2007-2008, with a slight slowdown in 2009. This is problematic to the extent that changes in growth differentially affect different regions of the income distribution. However, it will be shown that the identification strategy presented allows us to control for business cycle effects. A final confounding factor consists of the lagged effects of the 2004 reform, which introduced the option of the linear business schedule. The first year in our dataset, 2004, is also the first year following the introduction of the linear tax. In 2005 we see a year-on-year increase in the proportion of individuals filing the linear tax, and thereafter the take-up rate for this tax stabilizes in 2006 and 2007. Thus, we will exclude 2005 from our analysis of extensive-margin transitions.

A possible confounding event to the 2009 reform was a cut in the level of social security contributions for the financing of the disability insurance programme which occurred in 2007-2008. This is a contribution paid proportionately on employment income, but is a fixed rate for self-employed individuals approximately equivalent to the rate paid by those employed full-time on the minimum wage.[5] If we treat these social security contributions as a tax, this change would have caused a net increase in the tax gains from self-employment relative to employment at the lower end of the income distribution in 2007-2008 (and would have caused a net fall in the tax gains higher

[5] There is also a preferential rate of 30% of the minimum wage, which is applicable for the first 24 months after starting a business.

Table 1: Marginal tax rates and tax bands (in zł).

Marginal tax rate	2007	2008	2009-12
Tax-free amount (0.00)	0-3,015	0-3,091	0-3,091
0.18			3,091-85,528
0.19	3,015-43,405	3,091-44,490	
0.30	43,405-85,528	44,490-85,528	
0.32			>85,528
0.40	>85,528	>85,528	

up in the income distribution). On the extensive margin, our approach is to use the self-employed subject to progressive taxation as a control group for the self-employed subject to the linear schedule, or alternatively to examine changes along a band around a tax kink. This ought to control for the social security changes, as both groups would have been similarly in 2007-2008. On the intensive margin, the main identification strategy for the effect of 2009 tax reforms relies on assignment into treatment/control based on spousal income, and controls for base-year income. Again, this should control for the effects of the 2007-2008 social-security changes, which were independent of spousal income.

It is important to note that the transition from employment to self-employment and linear taxation was subjected to some restrictions. For instance, a year had to elapse between a taxpayer being employed by a company, and subsequently being hired by that same company as a business-owner. No such requirement operated in the reverse direction. From this, it should be clear that the costs of switching tax bases are not an exogenous parameter, but are a function of government policy. For the purposes of the present paper, we largely abstract away from this issue.[6]

[6]The question of how optimal tax policy could in principle be affected if individual-specific fixed costs could be influenced by e.g. changes in reporting requirements is left for future work.

3 Motivation

A central feature of the 2009 reform was that it altered the relative incentives to file income under the linear business schedule and the progressive (business and employment) schedules. In this section, we outline a theoretical model of reporting behaviour which describes how intensive elasticities interact with extensive-margin income-shifting responses in determining (1) the marginal deadweight loss (DWL) of tax reforms in a linear and piecewise linear context and (2) optimal taxation under fully non-linear tax schedules. Based on the predictions of this model, we will arrive at the empirical statistics sufficient for the calculation of the DWL and optimal tax formulae.

3.1 DWL under Linear Taxation

In the model, we postulate the existence of two tax bases subject to linear taxation, as well as a fixed costs of reporting self-employment income for taxpayers. We assume there exists a mass of individuals, each characterized by a parameter θ influencing the marginal cost of declaring an extra unit of taxable income of either type, and a parameter ϕ, which is the fixed cost of declaring business income instead of employment income. The cumulative distribution functions in the population are $F(\theta)$ and $G_\theta(\phi)$, with marginal densities $f(\theta)$ and $g_\theta(\phi)$, where the distribution of fixed costs is allowed to depend on θ. Individuals are assumed to have a quasi-linear utility of the form

$$u(c, l, b; \theta) = c - 1\{b = 0, l > 0\} \cdot \psi^L(l/\theta) - 1\{b > 0, l = 0\} \cdot (\psi^B(b/(\tilde{\omega}\theta)) - \phi)$$

where c is consumption, b is the amount of business income declared, l is the amount of labour income declared, and $1\{b = 0, l > 0\}$ as well as $1\{b > 0, l = 0\}$ are dummy variables equal to 1 if, respectively, any positive employment or self-employment income are declared. The function $\psi^K(.)$, where $K \in \{L, B\}$ indicates the tax-base, is convex and reflects the intuition of increasing marginal costs of producing an extra unit of taxable income as taxable income increases. It is linked to the elasticity of taxable income, and since previous studies suggest that the elasticities of taxable income are different for employment and self-employment, these are allowed to be different in the above model. The fixed cost ϕ can be interpreted as the cost of setting up a business, as well as attitudes towards risk or preferences towards self-employment. The quasi-linear utility implies an absence of income effects, and has been previously used by Diamond (1998).[7]

In this simplified model, individuals choose between employment and business income, but cannot report both (in the Appendix to Chapter 2, we relax this assumption in the context of a fully non-linear optimal tax problem). The budget constraint for each individual is just $c \leqslant (1 - \tau_L)l$ if the individual earns employment income, and $c \leqslant (1-\tau_B)b$ if the individual earns business income. On the intensive margin, conditional on being in a tax base, individuals will choose to report the level of income $1 - \tau_K = \psi'^K(k/\theta)/\theta$, where $k \in \{l, b\}$ denotes the level of income declared in a tax base. The solution to this yields the reported income supply functions $l(\theta)$ and $b(\theta)$. We may additionally define the indirect utility for each tax base, excluding fixed costs, as

$$v^K(\theta) = (1 - \tau_K)k(\theta) - \psi^K(k(\theta)/\theta).$$

An individual of type θ chooses the business tax base if the gain in indirect utility relative to the employment tax base exceeds the difference in associated fixed costs. The tax base choice for the individual is therefore determined by whether or not their fixed costs exceed the following threshold:

$$\widetilde{\phi}(\theta) = v^B(\theta) - v^L(\theta)$$

Consequently, the proportion of individuals of type θ reporting in the business tax base is simply $G_\theta(\widetilde{\phi})$, and the proportion in the employment tax base is simply $1 - G_\theta(\widetilde{\phi})$.

Let us suppose that the government increases the marginal tax rate on labour income τ_L by a small amount $d\tau_L$, with no change in the marginal tax

[7] Empirical analysis in the latter part of this paper suggests that income effects are in fact quantitatively small in the sample of taxpayers under consideration.

rate on business income τ_B. The reform has two effects on tax revenue. The first is a "mechanical" increase in tax revenue from employees as a result of taxpayers with wage income facing a higher tax rate. The total size of this effect is

$$dM = \int_{\Theta} \left[(1 - G_\theta(\widetilde{\phi}))l(\theta) \right] dF\theta \times d\tau_L \tag{1}$$

This is the projected increase in tax revenue from all taxpayers with employment income, absent any behavioural response.

The second effect of the reform is to cause a behavioural response which changes the reported income of taxpayers in the employment tax base. In the model proposed here, the behavioural response will have both an intensive-margin and an extensive-margin component. The intensive-margin response is the one traditionally emphasized in the ETI literature (see for instance Saez et al. (2012)): the change $d\tau_L$will induce a taxpayer of type θ to reduce the level of reported employment income by $\frac{\partial l(\theta)}{\partial \tau_L} d\tau_L$. This can be re-expressed as $-\frac{l(\theta)}{(1-\tau_L)}\varepsilon_L(\theta)d\tau_L$, where $\varepsilon_L(\theta) = \frac{1-\tau_L}{l(\theta)}\frac{\partial l(\theta)}{\partial(1-\tau_L)}$ is the standard elasticity of employment income for individual θ with respect to the marginal tax rate on employment income.[8] The total amount of income lost through this response is the integral of the change in tax revenue for all employed individuals: $\left(\frac{\tau_L}{1-\tau_L}\right) \int_{\Theta} \left[(1 - G_\theta(\widetilde{\phi}))l(\theta)\varepsilon_L(\theta) \right] dF\theta \times d\tau_L$.

However, for a taxpayer of type θ, the tax reform will also increase the threshold value of fixed costs at which it becomes optimal to switch from employment to business. This will induce a proportion of individuals with fixed costs below this value to switch to the business tax base. Specifically, the threshold $\widetilde{\phi}_\theta$ will change by

$$\frac{\partial \widetilde{\phi}_\theta}{\partial \tau_L} d\tau_L = -\frac{\partial \left(v^B(\theta) - v^L(\theta) \right)}{\partial(1-\tau_L)} d\tau_L = l(\theta)d\tau_L.$$

We may note that the change in the threshold for switching is therefore proportional to the change in the total quantity of tax paid on the employment income declared by an individual. The density of individuals of type θ who are induced to switch as a consequence of the reform is given by $g_\theta\left(\widetilde{\phi}\right) \times l(\theta)d\tau_L$.

[8]Since we assume away income effects, this is both the compensated and uncompensated elasticity.

For each individual in employment who switches to the business tax base, the net loss in tax revenue will be equal to $\Delta T^{L,B}(\theta) = \tau_L l(\theta) - \tau_B b(\theta)$. Hence, for type θ, the amount of revenue lost on the extensive margin is the density of individuals induced to switch, multiplied by the difference in total tax rates between the tax bases across which they are switching: $g_\theta\left(\widetilde{\phi}\right) l(\theta)\Delta T^{L,B}(\theta) \times d\tau_L$. This term is novel, and captures the fiscal externality arising out of the income-shifting occurring on the extensive margin. It is a weighted average over all types θ by the probability density of each type, $f(\theta)$, and is increasing in the density of individuals induced to switch tax bases, $g_\theta\left(\widetilde{\phi}\right) l(\theta)$, as well as the difference in the total tax burden between the two tax bases, $\Delta T^{L,B}(\theta)$.

Summing up both the intensive and extensive margin, the total change in tax revenue due to the behavioural response is equal to

$$\begin{aligned} dB &= \tau_L \int_\Theta \left[(1 - G_\theta(\widetilde{\phi}))\frac{\partial l(\theta)}{\partial \tau_L}\right] dF\theta \times d\tau_L - \\ &- \int_\Theta \left[g_\theta\left(\widetilde{\phi}\right) l(\theta)\left(\tau_L l(\theta) - \tau_B b(\theta)\right)\right] dF\theta \times d\tau_L = \\ &= -\left(\frac{\tau_L}{1-\tau_L}\right)\bar{\varepsilon}_L \times d\tau_L - \int_\Theta \left[g_\theta\left(\widetilde{\phi}\right) l(\theta)\Delta T^{L,B}(\theta)\right] dF\theta \times d\tau_L \end{aligned} \tag{2}$$

where we define $\bar{\varepsilon}_L = \int_\Theta \left[(1 - G_\theta(\widetilde{\phi}))l(\theta)\varepsilon_L(\theta)\right] dF\theta$ as the aggregate elasticity of employment income to the marginal tax rate on employment, weighted by the level of employment income. The term $\int_\Theta \left[g_\theta\left(\widetilde{\phi}\right) l(\theta)\Delta T^{L,B}(\theta)\right] dF\theta$ is the extensive-margin response. The total change in tax revenue dR due to the tax reform is just the sum of the mechanical and behavioural responses:

$$\begin{gathered} dR = dM + dB = \\ = \left\{\int_\Theta \left[(1 - G_\theta(\widetilde{\phi}))l(\theta)\right] dF\theta - \left(\tfrac{\tau_L}{1-\tau_L}\right)\bar{\varepsilon}_L\right\} \times d\tau_L - \\ - \int_\Theta \left[g_\theta\left(\widetilde{\phi}\right) l(\theta)\Delta T^{L,B}(\theta)\right] dF\theta \times d\tau_L \end{gathered} \tag{3}$$

The two terms accounting for the behavioural response are exactly equal to the marginal deadweight burden of the increase in the tax rate, provided the tax change is small. This is a well-known result of the envelope theorem: due to the optimizing behaviour of taxpayers, the behavioural response to a small tax change creates no additional welfare loss aside from the mechanical effect.[9] In the context of the above model, this is true both of responses at the extensive margins as well as the intensive margin. Individuals induced to switch to the business tax base are at the point of indifference between the two tax bases, and so incur no additional welfare loss due to switching. This is what allows us to measure the utility loss of the tax change in monetary terms purely in terms of the mechanical effect dM. Since tax revenue collected is $dR = dM + dB$, which is smaller than the utility loss dM as a result of the reform, the difference between the two, $-dB$, represents the extra amount lost in utility over and above the revenue collected.[10]

The same analysis can be performed for the effects of a small increase in the business marginal tax rate $d\tau_B$, with analogous terms reflecting the intensive and extensive-margin behavioural responses. The marginal deadweight burden of an increase in the business tax rate would now be increasing in the elasticity of business income $\bar{\varepsilon}_B$.[11]

At the optimum, absent distributional preferences, the policymaker would

[9]It is important to note that the lack of welfare loss due to behavioural responses only strictly holds if the tax change under consideration is infinitesimally small. With larger changes in tax rates, behavioural responses may indeed result in changes in utility, and in such cases the DWL forumlae above can only be viewed as a (linear) approximation around the original existing policy. Even if not viewed as approximations to welfare loses, e.g. if the tax reforms are large, as in Poland in 2009, the formulae above can still be useful since they represent estimates of the *actual* changes in tax revenue as a result of tax reforms. The analysis in Section 7 can be viewed in these terms.

[10]The revenue-maximizing tax rate can also be obtained from the above formula. This would be the one in which, at the margin, the mechanical effect from raising the marginal tax rate would be exactly offset by the loss in revenue due to behavioural responses, i.e. $dM = -dB$.

[11]Specifically, the formula showing the change in revenue due to a small increase in the business tax rate would be

$$dR = \left\{ \int_\Theta \left[G_\theta(\widetilde{\phi}) b(\theta) \right] dF\theta - \left(\frac{\tau_B}{1-\tau_B} \right) \bar{\varepsilon}_B + \int_\Theta \left[g_\theta \left(\widetilde{\phi} \right) b(\theta) \Delta T^{L,B}(\theta) \right] dF\theta \right\} \times d\tau_B \ .$$

Presuming the marginal tax rate on employment income is initially higher, there is a difference in that the extensive-margin behavioural response is positive, i.e. attenuates the DWL. This is because with an initially higher tax rate τ_B, switching away from self-employment towards employment generate a positive fiscal externality.

attempt to equalise the marginal DWL between the two tax bases. If the elasticity of taxable income is higher for self-employment than for employment (as is found by some studies, such as Kopczuk (2015) and Saez (2010)), and there were no extensive-margin responses, the DWL formulae would imply that the marginal tax rate should be lower on the business-income tax base than on the employment tax base. However, the above analysis also implies that this motive would have to be tempered by the degree of switching on the extensive-margin as a result of the arising differential in total tax rates between the two tax bases.

The above analysis demonstrates that the presence of an extensive-margin response increases the size of this additional deadweight burden relative to a model with just an intensive-margin response.[12] It also suggests which statistics are sufficient to evaluate the formula for DWL. These would be the size of the aggregate tax elasticity of the employment tax base, corresponding to the aggregate intensive-margin response to the reform, as well as an estimate of the total mass of taxable income lost as a result of switches on the extensive margin (i.e. the entire third term in equation (3)).

3.2 Estimating DWL of 2009 Reform

Having seen how the extensive-margin transitions affect the DWL of taxation under a simplified linear tax schedules, we can use an equation analogous to (3) to approximate the DWL for actual tax reforms such as the 2009 Polish tax reform. This could be the basis of comparison with welfare losses

[12] Piketty and Saez (2012) argue that extensive-margin responses do not alter the standard formulae for DWL. This is true in their model if the extensive-margin responses are between reporting some taxable income and reporting none. However, the main empirical studies relied on in estimating DWL of taxation rely on estimate ETI estimates (Gruber and Saez (2002), and Kopczuk (2005), being notable examples) tend to employ a panel method, such that only individuals who continue to report a positive level of taxable income in the personal income tax base. Individuals who switch to other tax bases, such as the corporate tax base, are excluded from the analysis. The model in this paper implies that this should exaggerate the size of the income loss, which would be attenuated since individuals who switch are continued to be taxed at the rate τ_B.

associated with other potential reforms which would have raised the same amount of revenue. It is important to note that the expressions for DWL here are at best an approximation, since the reform was large enough for the results of the envelope theorem to no longer apply. At the same time, the expressions capture behavioural revenue effects of the reform, which are of interest independently of DWL.

To approximate the effects of the type of reform introduced in Poland in 2009, we take into account the piecewise linear nature of the initial tax schedule, as captured by the presence of tax bands. Let $\bar{l}^1$and $\bar{l}^2$ be the threshold values for entering the middle and top tax bracket respectively, and $d\tau_L^1$ and $d\tau_L^2$ are changes to the marginal tax rate in these tax brackets. If we assume that the elasticity of taxable income for the employed is constant across the population, such that $\bar{\varepsilon}_L = \int_\Theta \left[(1 - G_\theta(\widetilde{\phi}))l(\theta)\varepsilon_L(\theta)\right] dF\theta = \varepsilon_L \times \int_\Theta \left[(1 - G_\theta(\widetilde{\phi}))l(\theta)\right] dF\theta$, the DWL associated with a small tax reform now is given by:[13]

$$\begin{aligned} DWL = \varepsilon_L &\left[\left(\frac{\tau_L^1}{1-\tau_L^1}\right)\bar{l}^1 \times d\tau_L^1 + \left(\frac{\tau_L^2}{1-\tau_L^2}\right)\bar{l}^2 \times d\tau_L^2\right] + \\ &+ \int_\Theta \left[\mathbf{1}\{l(\theta) \in [\bar{l}^1, \bar{l}^2]\} \times g_\theta\left(\widetilde{\phi}\right)\left((l(\theta) - \bar{l}^1) \times d\tau_L^1\right)\Delta T^{L,B}(\theta)\right] dF\theta + \\ &+ \int_\Theta \left[\mathbf{1}\{l(\theta) \in [\bar{l}^2, +\infty)\} \times g_\theta\left(\widetilde{\phi}\right)\left((\bar{l}^2 - \bar{l}^1) \times d\tau_L^1\right)\Delta T^{L,B}(\theta)\right] dF\theta + \\ &+ \int_\Theta \left[\mathbf{1}\{l(\theta) \in [\bar{l}^2, +\infty)\} \times g_\theta\left(\widetilde{\phi}\right)\left((l(\theta) - \bar{l}^2) \times d\tau_L^2\right)\Delta T^{L,B}(\theta)\right] dF\theta. \end{aligned} \tag{4}$$

Here, $\mathbf{1}\{l(\theta) \in [\bar{l}^1, \bar{l}^2]\}$ is an indicator for an individual falling into the middle tax bracket, $\mathbf{1}\{l(\theta) \in [\bar{l}^2, +\infty)\}$ is an indicator for an individual falling into the top tax bracket, and τ_L^1 and τ_L^2 are the marginal tax rates in the middle and top tax brackets respectively. The expression $\bar{h}^k$ is the average income in bracket k, multiplied by the number of individuals in that tax bracket. The expressions for the extensive-margin response now take into account how the reforms affect the probability of switching in a non-linear fashion, depending on whether an individual falls in the middle or higher tax bracket. The

[13]In Section 5, it is shown that the assumption of constant elasticities across the income-distribution appears to be in fact plausible.

probability of switching is simply the density of individuals at the point of indifference between the two tax bases, multiplied by the change in the income differential between the employment and business tax base. This is just $g_\theta\left(\widetilde{\phi}\right)\left((l(\theta)-\bar{l}^1)\times d\tau_L^1\right)$ and $g_\theta\left(\widetilde{\phi}\right)\left((l(\theta)-\bar{l}^2)\times d\tau_L^2+(\bar{l}^2-\bar{l}^1)\times d\tau_L^1\right)$ for each type θ, for the middle and top bracket, respectively.

It is well-known that intensive-margin elasticities ε_L and ε_B, as well as the average income levels in the tax bands, matter for DWL. It is important to note that now what also matters is the change in the probability of reporting employment income captured in the integral terms in (4). This is a statistic which can be estimated empirically. The formula in (4) also illustrates that in calculating the switches as a result of the reform, we ought to weight observations by the predicted differential in tax rates between the employment and business for each individual.

3.3 Optimal Non-linear Taxation Formulae

It is also illuminating to see how the presence of extensive-margin responses affect fully non-linear socially optimal tax schedules in the spirit of Diamond (1998) and Saez (2001). To do so, it is necessary to augment efficiency considerations with social preferences. For this purpose, we introduce social welfare weights $\widetilde{g}_\theta(\phi)$ and $\widetilde{f}(\theta)$, which may differ from the actual densities in the population and can be chosen to represent arbitrary preferences for redistribution between individuals of type θ and ϕ.

The statement of the social planner's problem and its solution is shown in detail in Appendix B. Here, we only restate the first-order conditions characterizing the solution to the social planner's problem, which describe the properties which need to be satisfied by the tax schedule for an incremental change in the value of the θ parameter in the population. For individuals reporting only employment income, this is

$$\frac{T'^{L}(\theta)}{1-T'^{L}(\theta)} = \left[\frac{1+\frac{1}{\varepsilon_L}}{\theta f(\theta)(1-G_\theta(\widetilde{\phi}))}\right] \times$$
$$\times \int_{\underline{\theta}}^{\theta} \left[\left\{\widetilde{f}(\hat{\theta})(1-\widetilde{G}_\theta(\widetilde{\phi})) - f(\hat{\theta})(1-G_\theta(\widetilde{\phi}))\right\} - f(\hat{\theta})\left(g_\theta(\widetilde{\phi})\Delta T^{L,B}(\hat{\theta})\right)\right] d\hat{\theta} \tag{5}$$

for all θ, where $T'^{L}(\theta)$ is the derivative of the employment tax schedule with respect to type θ, densities with $\tilde{}$ superscripts denote welfare weights and other variable definitions are as before. An analogous expression can be obtained for the business tax base:

$$\frac{T'^{B}(\theta)}{1-T'^{B}(\theta)} = \left[\frac{1+\frac{1}{\varepsilon_B}}{\theta f(\theta)G_\theta(\widetilde{\phi})}\right] \times [\int_{\underline{\theta}}^{\theta} \left\{\widetilde{f}(\hat{\theta})\widetilde{G}_\theta(\widetilde{\phi}) - f(\hat{\theta})G(\widetilde{\phi})\right\} d\hat{\theta}+$$
$$+ \int_{\underline{\theta}}^{\theta} f(\hat{\theta})\left(g_\theta(\widetilde{\phi})\Delta T^{L,B}(\hat{\theta})\right) d\hat{\theta}] \tag{6}$$

for all θ, where $T'^{B}(\theta)$ is the derivative of the employment tax schedule with respect to type θ.

From equation (5), it can be seen that the intensive-margin elasticities matter for both the DWL and optimal tax calculations. It can also be seen that, for the extensive margin, the relevant estimable parameters for the DWL and the optimal tax formulae are somewhat different. In particular, for the DWL calculation, we require an estimate of all of the transitions to the business tax base as a result of the tax reform, weighted by the tax differential between the bases. For the optimal tax calculations, however, what is required is an estimate of the probability of transitions as a result of a small change in the tax schedule. The goal of the remaining empirical sections of this paper is to estimate these estimable parameters, relevant for both DWL and optimal tax calculations.

4 Data

The dataset used for the empirical work is a proprietary dataset obtained from the Polish Ministry of Finance. It comprises a balanced panel of about a million Polish taxpayers over the years 2004 to 2012. This is a random sample selected out of the population of all taxpayers who reported any taxable income at least once in this period, and as a result is subject to attrition and replacement. The dataset contains information on individuals and spouses filing according to the progressive schedule, as well as self-employment individuals who chose to file self-employment income according to the linear schedule. In the latter case, individuals are legally prevented from filing jointly with a spouse and, as a result, we are unable to predict whether or not they have a spouse in practice. As with most tax return data, demographic information is limited, and contains only the taxpayer's age and gender. The number of children can be inferred only to the extent that the taxpayer claims deductions only available to those with children. Any deductions claimed by the individuals can be observed, as can be their place of residence by "voivodeship" (analogous to county-level).

The dataset allows us to link individuals who report both linear self-employment income and some employment income, and also to observe whether they report either self-employment or employment income, or both, if they report under the progressive schedule. Since less than a third of business owners report both employment and business income, for the empirical section we consider these individuals to be members of the self-employment tax base, on the grounds that they bear the fixed cost of participating in this tax base. An extension of the empirical study to estimate intensive-margin cross-elasticities, which would be relevant if we included individuals who report positive levels of both types of income, is left for future study.

As a preliminary exercise, it is illustrative to look at the share of individuals who declared business ownership at each level of income before and after the reform of 2009. This is done in Figure 2. First, it should be noted that rates of self-employment are extremely high in Poland for the highest income earners, rising from around 10% at the level corresponding to the first tax kink to over 50% at the very top of the distribution. In comparison, data presented in Scheuer (2012), based on the 2007 Survey of Consumer Finances and a similar definition of self-employment as that used in this paper, suggest that self-employment rates at the top of the income distribution amount to only around 11% in the United States. Secondly, comparing the pre-reform years 2005 and 2006 with the post-reform years 2010 and 2011, we observe a distinct fall in the share of individuals declaring self-employment income in the income range which corresponded to the middle tax bracket before 2009. This could be seen as *prima-facie* evidence that the reforms partially resulted in an extensive-margin adjustment between the self-employment and employment tax bases, with individuals transitioning away from self-employment and into employment due to the reduction in tax rates in employment. However, this change in share could have feasible also been due to purely intensive-margin responses. As individuals in employment at the top end of the income distribution experienced a cut in marginal rates in 2009, with the majority of self-employed individuals at the top end filing according to the (unchanged) linear schedule, the distribution of employment income may have shifted outwards, resulting in a share in the fall of income at each income level. As a result, we proceed to examine econometrically to what extent responses to the 2009 reform can be decomposed into intensive-margin responses on the one hand, and extensive-margin shifts on the other.

5 Intensive-Margin Responses

Section 3 explained how intensive-margin elasticities and extensive-margin parameters interact in formulae for DWL and optimal non-linear taxation, while Section 4 presented preliminary evidence on responses to the 2009 reform. In this section, we employ the variation which arose as part of the Polish 2009 reforms to estimate responses on the intensive-margin and evaluate the relevant intensive-margin elasticities.

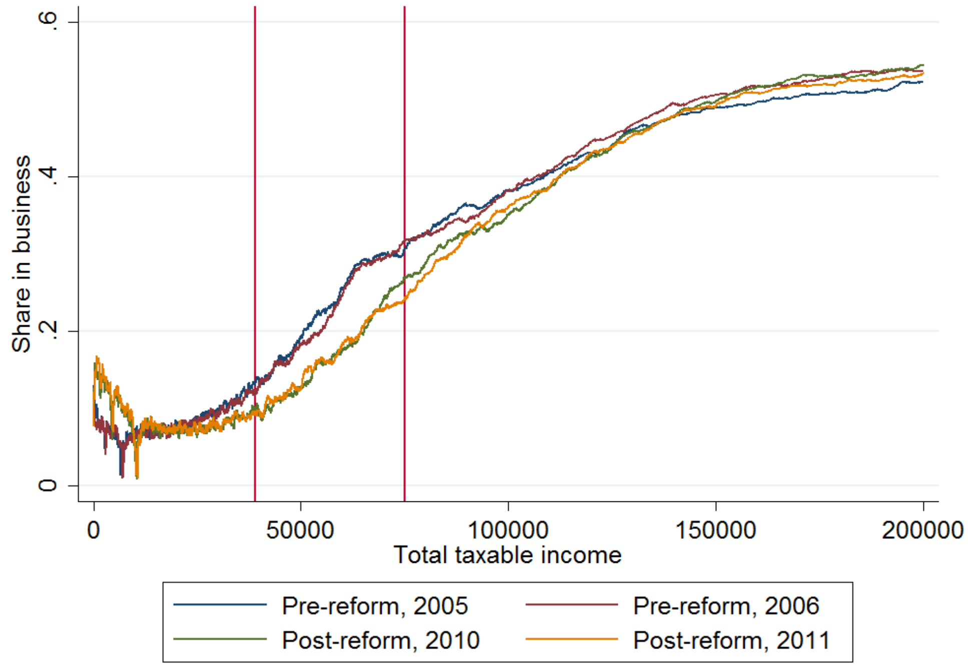

Figure 2: Share of self-employed individuals for each level of taxable income, pre and post-reform. The x-axis corresponds to taxable income in 2004 values.

5.1 Empirical Strategy

The primary outcome of interest is the log-change in taxable income as a function of changes in marginal tax rates. Following much of the existing studies, the panel regression specifications for individuals reporting employment income and business income are, respectively:

$$\Delta \log z_{it}^{L} \;=\; \varepsilon^{L} \Delta \log (1-\tau_{it}^{L}) \;+\; \eta^{L} \Delta \log [z_{it}^{L} - T_{it}(z_{it}^{L})] \;+\; f_{t}^{L}(z_{it}^{L}) \;+\; \Delta \xi_{it}^{L}$$

$$\Delta \log z_{it}^{B} \;=\; \varepsilon^{B} \Delta \log (1-\tau_{it}^{B}) + \eta^{B} \Delta \log [z_{it}^{B} - T_{it}(z_{it}^{B})] + f_{t}^{B}(z_{it}^{B}) + \Delta \xi_{it}^{B}$$

The variable z_{it}^{k} is reported taxable income in tax base k, $1-\tau_{it}^{k}$ is the net-of-marginal tax rate in tax base k, where $k \in \{L, B\}$ reflects the tax base under consideration. Here, $\Delta \log x_{it}^{k} = \log (x_{i,t+s}^{k}/x_{it}^{k})$ is the log-change in each of the respective variables between base-year t and year $t+s$. Consequently, ε^{k} evaluates the percentage change in taxable income as a result of a 1% change in the net-of-tax-rate (i.e. the elasticity of taxable income). Although income effects are excluded from our theoretical specification, we include them in the empirical analysis and these are measured by the parameter η^{k}, which is the marginal effect of a change in the disposable income available to an individual after taxation, represented by the variables $z_{it}^{k} - T_{it}(z_{it}^{k})$.[14] In particular, we take the relevant measure of income to be a couple's income for taxpayers with spouses, rather than individual income. Thus, $z_{it}^{k} - T_{it}(z_{it}^{k})$ is the sum of the total level of disposable income, net of taxation, available to both taxpayer i and their spouse at time t, net of taxation.found in the tax data: the age of the filer, and gender, as well a constant.

The difference s will usually be taken to be one year, although longer differences will also be looked at to ensure that results are not driven by timing responses.

The differencing operation means that all time-constant variables are eliminated. However, mean-reversion continues to be a key concern when using panel data to estimate ETIs, and could in principle result in the error

[14]It is shown that these income effects are in fact of low magnitude, in accordance with existing studies.

term being correlated with the instrument. In line with much of the literature, we therefore include a control for base-year income in the specification, $f_t^K(z_{it}^K)$ (see, for instance, Gruber and Saez (2002) and Kopczuk (2005)). For employment income, this will be a flexible piecewise linear function of log of base-year taxable income. For business income, due to a smaller sample size, we will include a linear control in the log of base-year income.

Choice of instruments. As has been widely recognized in the literature, the tax variables are endogenous due to the progressivity of the tax schedule, individuals who experience positive income growth will face higher marginal tax rates, resulting in biased estimates of the elasticity parameter. The variable for the change in the net-of-marginal tax rate therefore needs to be instrumented. The instrument for $\Delta \log (1 - \tau_{it}^B)$ proposed here is a dummy equal to 1 if the income of the spouse is sufficiently high at time t for the couple to enter at least the second tax bracket under joint reporting, and 0 otherwise. The identification of the elasticities in this case comes from comparing the changes in taxable income for individuals who experienced the higher falls in the marginal tax rate associated with the middle and top brackets (for whom the instrument is equal to 1) with the changes in in taxable income of those in the bottom bracket who experienced a negligible fall (for whom the instrument is equal to 0). For any level of base-year taxable income z_{it}^k in the range under consideration, it is possible to find both an individual who is 'treated' and an individual who falls into the 'control' group.[15] For the intensive-margin estimates, all income variables are deflated using average taxable income growth in the population, thus accounting for changes in tax rates due to 'bracket creep'. The instrument for $\Delta \log [z_{it}^L - T_{it}(z_{it}^L)]$ is standard in the literature (see, for instance, Gruber and Saez (2002) and Kopczuk (2005)), and amounts to using the predicted change in the log of disposable income based on time t information, $\Delta \log [z_{it}^{L,P} - T_{it}^P(z_{it}^{L,P})]$, absent any change in behaviour. This essentially involves applying the predicted tax schedule in the year $t + s$ to (deflated) base-year income.

[15]This is possible for the income range $z_{it}^k \in [0, 2 \times \bar{z}^{(2)}]$, where $\bar{z}^{(2)}$ is the threshold for entering the middle bracket. Consequently, our estimates of ETIs are limited to this income range. However, this range spans approximately 95% of Polish taxpayers.

The identification strategy can be helpfully illustrated diagrammatically. This is done in Figure 3, which compares the tax schedules faced by individuals who have a spouse declaring the same level of income as they are (or, alternatively, are single – the tax schedules in this case are the same, if we presume tax liability is divided equally between the couple), and individuals with a zero-income spouse. As can be seen, all individuals in the middle tax bracket (43,405zł-85,528zł), for instance, who have a spouse at the same level of income experience a cut in the marginal tax rate (represented by the slope of the tax schedule), as well as a cut in the total tax liability, represented by the vertical distance between the new and the old schedules. On the other hand, individuals who have the same basic income but who have a spouse with zero income experience a negligible cut in marginal taxes in the same income range, and also a negligible cut in total tax liability. In this way, the year-on-year change in income for individuals with a low-income spouse in this income range could plausibly provide a counterfactual for individuals in the same income range, but who have higher-earning spouses. This is precisely the logic of the instrumental variable strategy presented above.

This identification strategy is importantly different from that used in Gruber and Saez (2002). Their approach would instead would use instruments based on membership of the highest two income brackets in the base year. In this alternative strategy, the counterfactual levels of income growth are provided in an important way from non-reform years – in particular, these are captured by the spline terms in the above regression framework. In terms of Figure 4, this would involve estimating a pattern of income growth for each income level on the x-axis using a flexible spline, and subsequently using this relationship and observed income growth in the group of individuals with income just below 43,405zł to provide the counterfactual for what the income change would have been above 43,405zł, absent reform. As will be shown, this strategy is not robust to changes in the pattern of income growth year-on-year, and may significantly bias estimates where stable income growth patterns are not observed.

Identifying assumption. The crucial identifying assumption here is that the spouse's income being low enough for the taxpayer to fall into the lowest

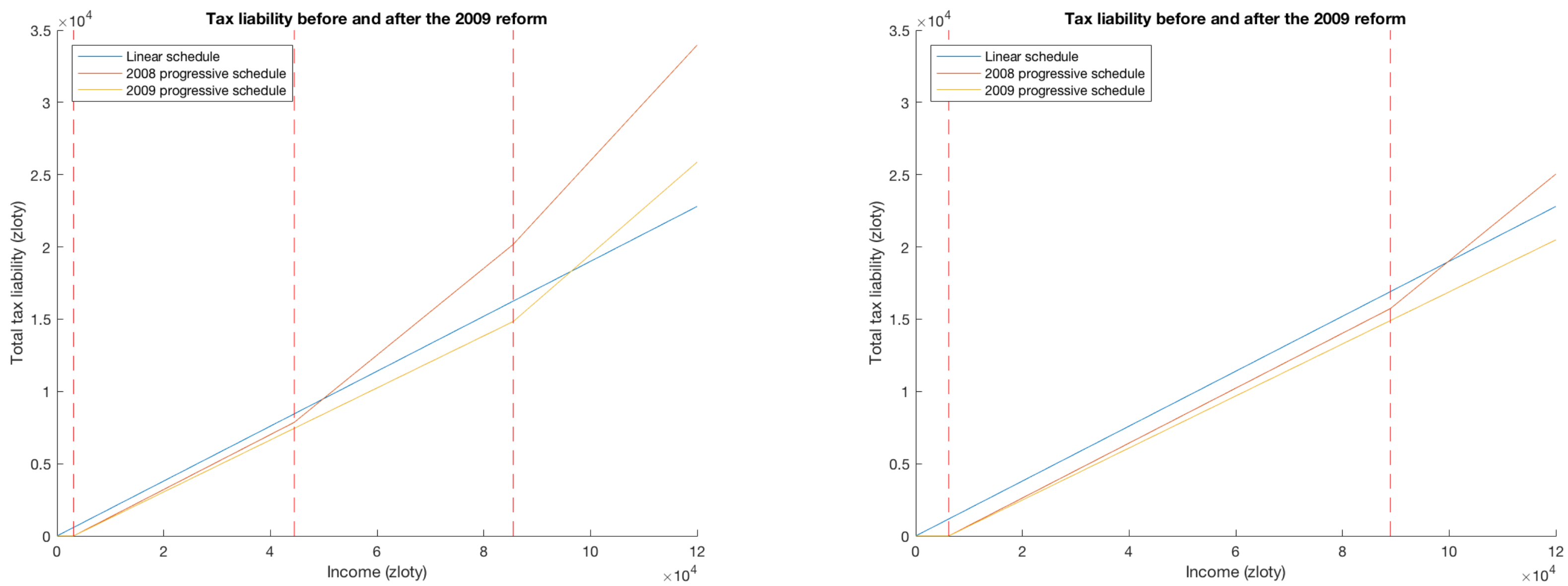

Figure 3: Effects of reform for individuals reporting who are: (i) single, or alternatively have a spouse declaring the same income level as the individual (left panel) and (ii) a spouse declaring zero income (right panel). The blue lines represent the linear self-employment schedule, which remains unchanged before and after the reform. The red line represents the progressive schedule (for employment or self-employment) before the reform, while the yellow line represents the progressive schedule (for employment or self-employment) after the reform. The x-axis represents nominal income, while the y-axis represents total nominal tax liability.

tax bracket is independent of unobserved variables driving the *change* in the taxpayer's taxable income, conditional on covariates.[16] This amounts to a common trends assumption which should hold in years outside of the reform, and it is possible to verify it empirically. To this end, Figure 4 plots the average level of income growth for the treatment and control groups of those reporting employment income for years preceding and following the reform, as well as the year of the reform itself. Both groups appear to show very similar patterns of income dynamics in the years outside of the reform for any given income level. In the year of the reform, however, the group pushed into the second and third brackets seems to have a discernibly higher rate of taxable income growth, and this appears to be relatively stable across the income distribution. Thus, it would be appear that the identifying assumption holds relatively well. Indeed, the graphs suggest that the methodology used in other ETI studies (Gruber and Saez 2002; Kopczuk 2005), which assumes a stable gradient of income growth from year to year outside of reform, would be questionable in the present case. For instance, the slope of the gradient of income growth in 2008 appears to be much steeper than that in 2010. Thus, the method would likely have produced a high elasticity estimate using the 2010 placebo, with 2008 as the baseline year. The method here allows us to sidestep this problem, as the patterns of income growth for the treatment and control groups are similar within each non-reform year, even if they differ from year to year.

Sample selection. Since mean-reversion is particularly acute at the very bottom of the distribution, we will exclude individuals with a base-year income below 20,000zł in 2004 terms from the sample of the employed, and a base-year income below 5,000zł in 2004 terms from the sample of business owners. As in Gruber and Saez (2002), we censor changes in log income at

[16]Importantly, this does not require the assumption that there is not correlation in *levels* of income between spouses. Indeed, there is considerable correlation between the incomes of spouses in the dataset. Theoretically, this could be explained by a model in which there is separability between the taxable income of the primary taxpayer and their spouse. Of course, in this setup there is still the possibility that an individual will respond to the level of a spouse's income through the income effect.

± 7. This means dropping individuals who experienced a thousand-fold rise or fall in taxable income from year to year. To maximize the sample size used in the regressions, we include both taxpayers and their spouses. No age data is available for spouses, and so the only control variable for age used in this case is the filer's age. For both tax bases, only individuals who report income in that tax base only are included in the sample – thus, all individuals who report some amount of both types of income are excluded. The same identification strategy is used both for the employment and business samples. This naturally relies on the sample of business owners who chose not to file according to the linear schedule.[17] Hence, we make the assumption that the selection of business owners into the linear tax base was independent of their elasticities of taxable income, and was driven by, for instance, preferences for deductions.

5.2 Results

The baseline results for the sample of individuals reporting employment income are shown in Table 2. The estimated elasticity of employment income ε^L without the inclusion of income effects is 0.218, and is highly statistically significant. The inclusion of income effects does not alter the estimate substantially. With income effects, it rises slightly to 0.233 and continues to be highly statistically significant. Estimated income effects η^L are statistically significant at the 5% level, and are small and positive in line with the existing literature (Gruber and Saez, 2002). All of the placebo estimates of the elasticity are statistically insignificant, supporting the validity of the exclusion restriction belonging to the treatment group only appears to affect income growth only in the year where a significant change in marginal tax rates occurred. Finally, the elasticity estimates based on a 2-year difference between 2007-2009 is larger at 0.653 with income effects, suggesting that the response to the reform is unlikely to have been driven by a timing response

[17] For self-employed individuals filing according to the linear schedule, joint filing with a spouse is not permitted, and moreover the marginal tax rates on business income faced by this group remained unchanged in 2009.

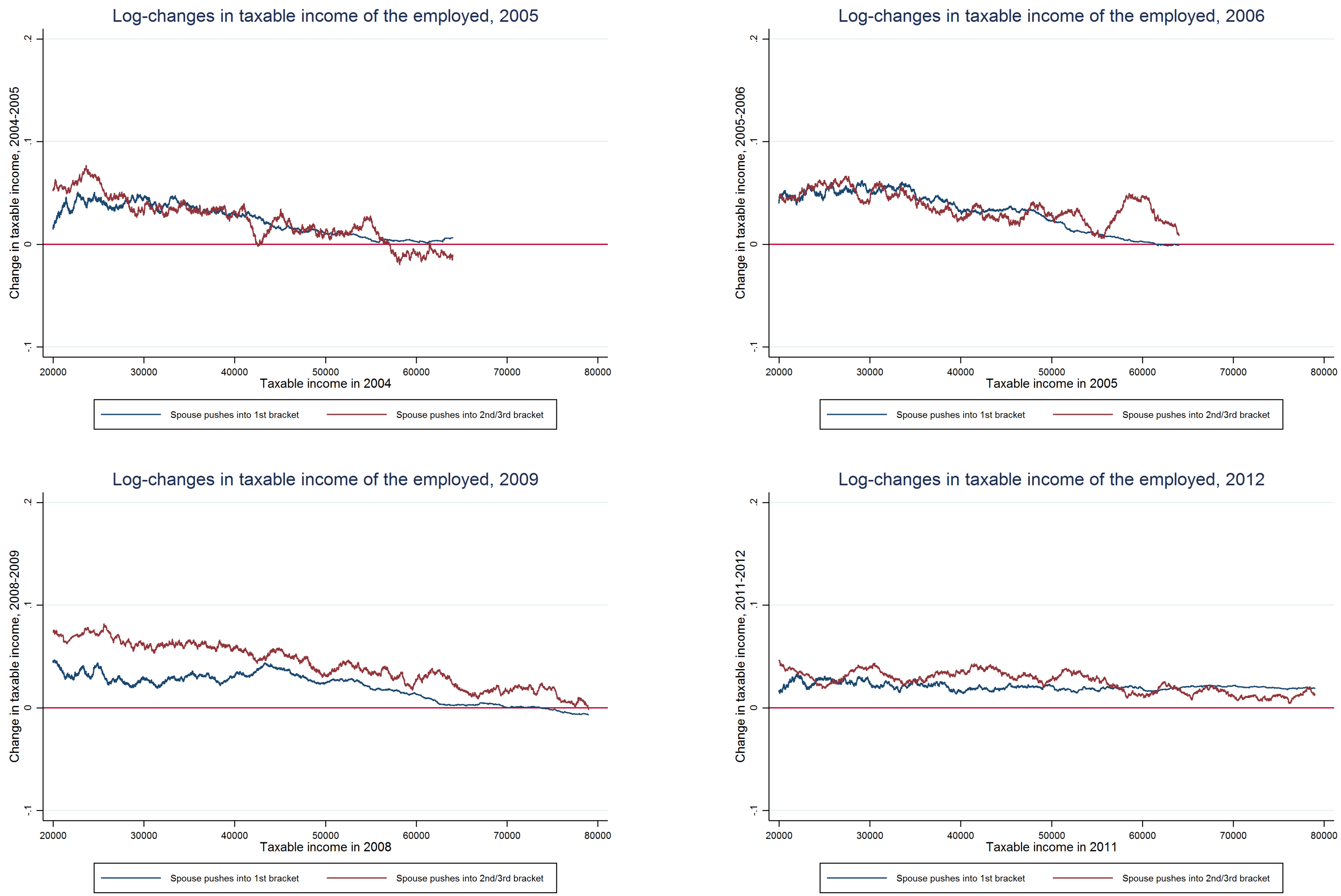

Figure 4: Taxable income growth rates for individuals whose joint filing with a spouse pushes them above the first tax kink (red line) and individuals who remain in the bottom tax bracket (blue line). The x-axis represents CPI-deflated income (2004 values), while the y-axis represents the log-change in taxable income.

(for instance, by taxpayers shifting income from 2008 to 2009).

The identification strategy used here, together with the large size of the sample of employed individuals, allows us to estimate the ETI for small subsets of the distribution of employment income. The results are presented in Table 3. Above the 20,000 zł band, the ETI estimates show surprising stability, and all fall between 0.207 and 0.251. It is worth noting that this spans most of the employment earnings distribution in the economy, with median income in Poland in 2008 being 31,680 zł.

The results of applying the methodology used by Gruber and Saez (2002) to the present sample is illustrated in the final column of Table 3. It can be seen that the result is an order of magnitude different from the baseline methodology used here. Given the variation in the profile of income growth already noted in Figure 4, this is not surprising, and is likely to reflect the variability of the income growth profile from year to year. Specifically, an estimate of the pattern income growth for the years 2004-2005, 2005-2006 etc., is used here to predict the non-reform pattern of income growth in 2008-2009. However, if this pattern is not constant, this can lead to either an over or an under-prediction of the actual counterfactual.

The results for the sample of business owners is presented in Table 4. It can be seen that although the sample of individuals is much smaller than for the employment sample, the estimated business elasticity is still statistically significant at the 5% level. At 0.657, it is estimated at three times the value of the baseline estimate for the employment ETI. There is more variation in the magnitude of the placebo estimates from year to year than in the employment sample, which could be expected given the smaller sample size. However, these never produce an ETI estimate significantly different from zero, which is what we would expect if the identification assumption holds. The inclusion of income effects decreases the ETI estimate to 0.492, and the estimated income effects are significantly larger than for the employment subsample at 0.11. As for the employment sample, a two-year difference increases the size of the estimate without income effects to 1.701, although this falls to 1.250 and loses statistical significance with the inclusion of income effects. Interestingly, this is close to the elasticity of 1.099 found by Kopczuk (2015)

Table 2: Own-elasticity estimates for employment sample.

Years	2008-09	2005-06 (Placebo)	2006-07 (Placebo)	2010-11 (Placebo)	2007-2009	2004-09 (Gruber-Saez)
A. Baseline						
ε_L	0.218***	-0.041	0.003	-0.008	0.614***	7.390***
	(0.025)	(0.033)	(0.037)	(0.098)	(0.063)	(1.170)
Number of observations	100,248	69,461	78,911	100,648	70,902	286,754
B. Incl. income effects						
ε_L	0.233***	-0.035	0.019	0.010	0.653***	–
	(0.026)	(0.033)	(0.037)	(0.040)	(0.170)	
η_L	0.017**	0.017***	0.013	0.028***	0.035	–
	(0.009)	(0.005)	(0.011)	(0.009)	(0.185)	
Number of observations	94,566	65,913	73,480	94,865	66,924	

Notes: All regressions include a 10-piece piecewise linear spline in the log of base-year income as a control. Instruments are as described in the text. The placebo estimates use the difference in net-of-tax rates from 2008-2009 for calculating the size of the effect in the placebo years (2005-06, 2006-07 and 2010-11). The sample consists of individuals who reported only employment income for the years spanned by each estimation, with all individuals who earned below 20,000zł in 2004 terms or above twice the first tax threshold minus 20,000zł.

Table 3: Elasticity estimates using the 2009 Polish tax reform for different parts of the income distribution.

	Employment					Self-employment	
Income interval (zł)	20k-25k	25k-30k	30k-35k	35k-40k	40k-45k	Bottom 50%	Top 50%
	0.177						
	(0.320)						
2008-09	0.218***	0.207***	0.252***	0.251***	0.207	0.071	0.845***
	(0.044)	(0.040)	(0.068)	(0.084)	(0.168)	(0.670)	(0.293)

Notes: All regressions include a 10-piece linear spline. Instruments are as described in the text. The self-employment sample is divided around the median level of income in the sample in the base year, and the elasticity estimates are given for the bottom 50% and the top 50%. The median level of income is 24,912zł in 2008 (in 2008 values).

for Polish business owners, and confirms that the ETI for business owners is likely to be significantly larger than that for employees.[18]

6 Extensive-Margin Responses

Having estimated the intensive-margin responses to the 2009 tax reform, we now turn to estimating the extensive-margin parameters identified in Section 3. The relevant dependent variable for estimating the size of the extensive-margin response to reform according to the model in Section 3 is the change in the probability of reporting any income in a given tax base. Based on a linear approximation around a starting threshold value of fixed costs $\widetilde{\phi}_\theta^{Old}$, we know that the change in the proportion reporting business income as a result of a tax reform is

$$\triangle G\left(\widetilde{\phi}_\theta\right)|_{\widetilde{\phi}_\theta=\widetilde{\phi}_\theta^{Old}}= G\left(\widetilde{\phi}_\theta^{New}\right) - G\left(\widetilde{\phi}_\theta^{Old}\right) \approx g\left(\widetilde{\phi}_\theta^{Old}\right)\left(\widetilde{\phi}_\theta^{New} - \widetilde{\phi}_\theta^{Old}\right)$$

From our model, we know that if a change in tax policy is small, it affects utility only through the direct effect on the change in the tax differential between tax bases. Here, we presume that this is a reasonable approximation in the context of the 2009 reform, and ignore second-order effects of changes in reporting behaviour on utility differentials between tax bases. Hence $\widetilde{\phi}_\theta^{New} - \widetilde{\phi}_\theta^{Old} = d(T_\theta^L - T_\theta^B)$, i.e. the size of the change in relative taxation for type θ. An empirical analogue of this proposed here is the linear probability model

$$\triangle Pr(base_{it} = L) = \alpha\triangle\left[T_t^L(z_{it}^L) - T_t^B(z_{it}^B)\right] + \varsigma_{it} \qquad (7)$$

where α is a coefficient corresponding to an estimate of the density $g\left(\widetilde{\phi}_\theta\right)$ around its original value, and ς_{it} is an innovation term capturing other factors which may induce an individual to change tax bases from year to year.

It is important to note how precisely the term α relates to the term $g\left(\widetilde{\phi}_\theta\right)$, which features in the formulae in Section 3. In principle, any estimate of

[18] Kopczuk's estimates are based on the 2004 reform which introduced the option of the flat tax for business owners. The estimate is from a specification involving three-year differences, and includes income effects, estimated at -0.116.

changes in the share of individuals reporting in the employment tax base is a *local* estimate, around the density of fixed costs at which individuals are indifferent. For the purpose of calculating the effects of the 2009 reform, the change in the share of individuals reporting within the employment tax base is precisely what is required. The main approximation used here will be that, as with the intensive-margin elasticities, the parameter is constant regardless of the level of income declared by any individual. However, for purposes of calculating an optimal tax schedule, a departure from the local estimated density may be necessary. This exercise is performed in a companion paper, Zawisza (2017), for a stylised economy based on estimates presented here.

Finding the simple change in the fraction of individuals reporting business income around the time of the 2009 reform may of course be inappropriate as an estimate of the effect of the reform, however, as the term ς_{it} may contain other factors driving switching which are correlated with change $\triangle\left[T_t^L(z_{it}^L) - T_t^B(z_{it}^B)\right]$. For instance, an individual who incurs a negative income shock in the employment tax base may face a lower tax liability, but may also be more likely to switch to self-employment. Instead, for identification of the effect, we focus on *transitions* occurring between the employment and business tax bases for which suitable treatment and control groups may be constructed. If we assume that an individual only has a choice between business and employment income, and there is no entry or exit into reporting any income at all, the following describes the relationship between the change in the fraction of employment income between t and $t+s$ and transitions between bases

$$\begin{aligned}\triangle Pr(base_{it} &= L) = \\ &= Pr(base_{it+s} = L|base_{it} = B) \times Pr(base_{it} = B) - \\ &\quad -Pr(base_{it+s} = B|base_{it} = L) \times Pr(base_{it} = L) = \\ &= Pr(trans_{it+s}^{B\to L}) \times Pr(base_{it} = B) - \\ &\quad -Pr(trans_{it+s}^{L\to B}) \times Pr(base_{it} = L). \end{aligned} \tag{8}$$

where $base_{it} = K$ means belonging to tax base K at time t, and $trans_{it+s}^{B\to L}$ refers to a transition from the business tax base to employment. Accordingly, we will separately estimate the two components $Pr(trans^{B \to L}_{it+s})$ and $Pr(trans^{L \to B}_{it+s})$ and use them to back out the total change in reporting probability $\triangle Pr(base_{it} = L)$.

Table 4: Own-elasticity estimates for the business sample.

Years	2008-09	2005-06 (Placebo)	2006-07 (Placebo)	2010-11 (Placebo)	2007-09
A. Baseline					
ε_B	0.657**	-0.321	-0.118	-0.062	1.701***
	(0.287)	(0.618)	(0.452)	(0.477)	(0.585)
Number of observations	6,856	6,291	6,327	7,225	6,393
B. Incl. income effects					
ε_B	0.492**	-0.465	0.156	-0.263	1.250
	(0.279)	(0.623)	(0.414)	(0.468)	(1.163)
η_B	0.111**	0.027	-0.009	0.148	-0.019
	(0.062)	(0.048)	(0.061)	(0.092)	(1.187)
Number of observations	6,269	5,848	5,853	6,548	5,865

Notes: The sample consists of individuals who reported only business income, with all individuals who earned below 5,000zł in 2004 terms, or above twice the first tax threshold minus 5,000zł. The log of base-year income is used as a control in all regressions. The instruments are as indicated in the main text.

6.1 Empirical Strategy

In our approach in this section, we estimate an equation of the form (7) for both $Pr(trans^{B \to L}_{it+s})$ and $Pr(trans^{L \to B}_{it+s})$. If the self-employment tax schedule does not change significantly, the change in relative taxation term $\triangle[T^L_t(z^L_{it}) - T^B_t(z^B_{it})]$ will be dominated by $\triangle T^L_t(z^L_{it})$. This is true in the context of the 2009 reform, for instance if we focus on individuals who file under the linear schedule (and did not experience a change in the schedule at all) or individuals filing under the progressive schedule with a low-income spouse, thereby avoiding falling into a higher tax bracket pre-reform (and experiencing a 1% fall in the marginal tax rate in both employment and self-employment schedules).[19] Specifically, we estimate linear probability models

$$Pr(trans_{it+s}^{L \to B}) = \alpha_1 \left[T_{t+s}^{L}(z_{it+s}^{L,P}) - T_t^{L}(z_{it}^{L}) \right] + \gamma_1 \mathbf{1}\{t \geqslant 2009\} + \beta_1 \mathbf{1}\{i \in T\} + \varsigma_{it}^{1}$$

$$Pr(trans_{it+s}^{B \to L}) = \alpha_2 \left[T_{t+s}^{L}(z_{it+s}^{L,P}) - T_t^{L}(z_{it}^{L,P}) \right] + \gamma_2 \mathbf{1}\{t \geqslant 2009\} + \beta_2 \mathbf{1}\{i \in T\} + \varsigma_{it}^{2}$$

where $z_{it+s}^{L,P}$ is predicted employment income in year $t+s$, $\mathbf{1}\{t \geqslant 2009\}$ is a dummy for a post-reform year and $\mathbf{1}\{i \in T\}$ is a dummy for belonging to a suitably defined treatment group. The difference s used here will be two years. This is to allow for individuals who transition gradually between tax bases, and to take into account the one-year grace period required by Polish tax authorities between being employed and providing services to the former employer as a business owner. A transition from employment to the business tax base is considered to have occurred if an individual has begun reporting some level of business income, where none had been reported previously. Symmetrically, a transition from the business tax base to the employment tax base is assumed to have occurred if an individual ceases to report any business income in the next period, but reports some employment income.[20]

Employment to business transitions. As has already been mentioned, the key variable of interest – the predicted change in the total tax liability in the employment tax base – is potentially endogenous and must therefore be instrumented. For the employment-to-business transitions, the identification strategy pursued here is analogous to the 'bracket creep' methodology of Saez (2003). In particular, we restrict attention to a band of income around the first kink in the tax-schedule pre-reform. The instrument for

[19]The component $\triangle T_t^B(z_{it}^B)$ is accounted by predicting the change in income and applying the predicted (unchanged) tax schedule to this new level of income. The predicted income will be inflated by average income growth in the intervening period, as in Gruber and Saez (2002). Based on the results of the Heckman selection model presented in Appendix A, we do not expect there to be significant changes in the level of declared taxable income on switching between tax bases.

[20]Thus, such an individual may continue to receive some employment income. The intention here is that such an individual must still bear the fixed cost of undertaking business activity.

$T^L_{t+s}(z^{L,P}_{it+s}) - T^L_t(z^L_{it})$ is then a dummy for having income predicted to fall above the threshold in year $t+s$. This is equivalent to assigning such individuals to a treatment group, while individuals whose income is predicted to fall below the threshold are assigned to the control group. The intuition behind this approach is that individuals below the cutoff are less likely to be significantly affected by the 2009 reform since their predicted income will not cross the first tax threshold beyond which there is a large change in total taxation. They are therefore unlikely to experience a rise in disposable income which could change the relative tax advantages of switching to the linear business schedule. However, they are assumed to be close enough in unobserved characteristics to the treated group that they constitute a suitable control group.[21]

To implement the strategy, the instrument for $T^L_{t+s}(z^{L,P}_{it+s}) - T^L_t(z^L_{it})$ is the interaction between the post-reform and treatment dummy:

$$\mathbf{1}\{t \geqslant 2009\} \times \mathbf{1}\{i \in T\}.$$

It can be easily shown that this results in a Wald estimator of the form

$$\alpha = \frac{\begin{gathered}\Big(\mathbf{E}[trans^{L\to B}_{it+s}|T] - \mathbf{E}[trans^{L\to B}_{it}|T]\Big) - \\ - \Big(\mathbf{E}[trans^{L\to B}_{it+s}|C] - \mathbf{E}[trans^{L\to B}_{it}|C]\Big)\end{gathered}}{\begin{gathered}\Big(\mathbf{E}[T(z^{L,P}_{it+s}) - T^L_t(z^L_{it})|T] - \mathbf{E}[T^L_t(z^{L,P}_{it}) - T^L_t(z^L_{it-s})|T]\Big) - \\ - \Big(\mathbf{E}[T(z^{L,P}_{it+s}) - T^L_t(z^L_{it})|C] - \mathbf{E}[T^L_t(z^{L,P}_{it}) - T^L_t(z^L_{it-s})|C]\Big)\end{gathered}}.$$

This compares the change in the rate of transitions in the treatment group to the change in the control group in the numerator. The denominator, on the other hand, compares the change in the predicted levels of taxation between the treatment group and the control group.

[21] The individuals in the treatment group, i.e. above the tax kink, are very likely to experience a change in the marginal tax rate. Since the bands around the tax kink are quite wide, however, many of those individuals are also likely to experience a non-trivial change in the total tax rate and, consequently, a non-trivial change in the relative tax difference between employment and self-employment.

The identifying assumption behind the estimate is that, absent reform, the difference in rates of transition between individuals immediately below and above the cutoff is stable. This assumption can be verified by examining the relative differences in transition rates for the treatment and control groups in years which did not involve tax reform. These patterns of transition over a two-year lag are demonstrated in Figure 5. It can be seen that, in the post-reform years (2010 and 2012), the patterns of transition were fairly stable around the first tax kink (although there appears to be a small surge in transitions in 2010 mid-way between the first and second tax kinks). Between 2008 and 2010, around the year of the introduction of the reform, however, we observe a significant fall in the level of transitions around the first kink. As has already been indicated, this fall appears to persist between 2010 and 2012.

Business to employment transitions. For transitions from business to employment, the instrument for an exogenous change in the difference in taxation between the tax bases is filing under a linear schedule, when compared to the population of individuals filing under a progressive schedule but with a sufficiently low-income spouse to fall into the bottom tax bracket. The intuition is that for those filing business income under a linear schedule, the differential between employment and business tax levels changed more as a result of the 2009 reform than for those who were already filing business income under a progressive schedule (indeed, with a low-income spouse, there occurred only a 1% change in the marginal tax rate). An individual owning a business under linear reporting should therefore be more likely to switch to employment than a business owner reporting under the progressive tax base with a low-income spouse. They are initially more likely to fall into the higher tax brackets on switching than an individual sharing tax liability with a spouse, all other things being equal; this becomes less of an issue post-reform due to the flattening of the progressive tax schedule. Figure 6 suggests that the parallel trends assumption between two groups seems to hold in the pre-reform years 2006-2008 as far as transitions are concerned.

Figure 5: Transitions from employment to business between base year t and year $t+2$ for individuals who do not report with a spouse, shown in the left panel for the years 2008 and 2010, and in the right panel for the years 2010 and 2012.

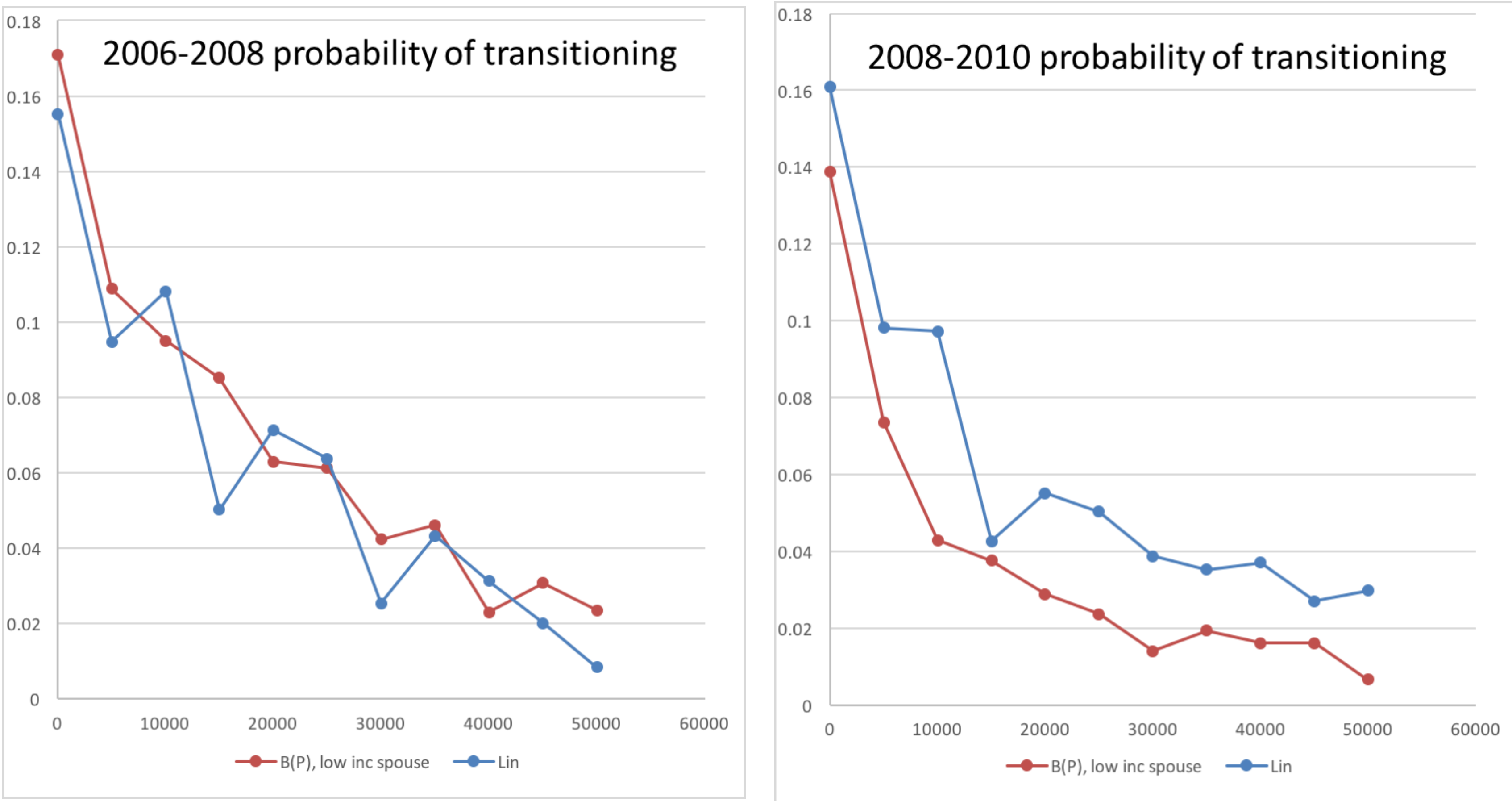

Figure 6: Transitions from business to employment between base year t and year $t + 2$ for individuals who file under the linear schedule in the base year, and individuals who file under the progressive schedule but report with a spouse whose income falls below the first tax threshold.

6.2 Results

The results for the extensive-margin regressions are presented in Table 5. Panel A presents the estimates for the employment-to-business transitions, while panel B presents estimates for transitions from business to employment.

Employment to self-employment transitions. The first line reports estimates based on comparing the years 2008 and 2010. The baseline estimate of $\alpha\times$ 10,000 zł at 0.0196 implies that a 10,000 zł increase in the tax burden induces 1.96 percentage points of those reporting employment income to transition to business income. A wider band around the second kink, which widens the definitions of treatment and control groups, lowers the estimate somewhat to 0.0162, or 1.62 percentage points in response to a 10,000 zł increase in the tax burden. The placebo estimates from comparing the years 2010 and 2012 are statistically insignificant, and thus do not contradict the identifying assumptions.

Self-employment to employment transitions. The estimate of $\alpha\times$ 10,000 zł of -0.0554 for transitions from business to employment shows that, for those who began in the business tax base, a 10,000 zł decrease in the tax burden in employment relative to business would induce 5.54% of those in business income to transition to employment. Again, the placebo estimates from the period from 2010 to 2012 are statistically insignificant, and thus support our identification strategy.

We may combine the estimated responses on transitions to and from self-employment, using formula (8). For simplicity, we may take the level of income of 150,000 zł, where there is roughly an equal share of individuals in employment and in self-employment. In this case, the total estimate of $g\left(\widetilde{\phi}_{\theta}\right)$ would be $0.5 \times (-0.0554) - 0.5 \times 0.0196$, i.e. 0.0375, based on the above estimates. In other words, a decrease in the differential between employment and self-employment of 10,000zł at this income level would result in a total reduction of 3.75% of individuals transitioning to employment, partly due to increased transitions from self-employment to employment, and partly because fewer individuals would transition from employment to self-employment.

Table 5: Extensive-margin responses.

	2008 and 2010	2010 and 2012 (Placebo)
A. Employment to business transitions		
30k zł band around first kink		
$\alpha^{\text{L to B}} \times$ 10,000 zł	0.0196**	-0.0081
	(0.00972)	(0.0103)
Number of observations	170,287	177,292
60k zł band around first kink		
$\alpha^{\text{L to B}} \times$ 10,000 zł	0.0162***	-0.0005
	(0.00464)	(0.0042)
Number of observations	375,771	388,985
B. Business to employment transitions		
$\alpha^{\text{B to L}} \times$ 10,000 zł	-0.0554***	-0.0071
	(0.0084)	(0.0051)
Number of observations	26,639	18,301

Notes: The bands concern the width of the tax kink at the entry to the second tax bracket, e.g. at 30k zł band means the sample is restricted to those 15k zl below the cutoff and above (the control group) and those 15k zł above the cutoff and below (the treatment group).

7 Revenue Impact and Deadweight Losses

Our estimates from the previous section can be used to estimate the changes in tax revenue as a result of the 2009 reforms in Poland due to changes in behaviour, and compare them to mechanical changes in tax revenue purely due to changes in rates, holding behaviour fixed.[22] The results are presented in Table 6, and are based on formula (4) presented in Section 3. On the intensive-margin, this involves calculating the total income levels in each bracket, and adjusting it by the relevant tax parameters and elasticity estimates. The calculation on the extensive-margin is more involved. Specifically, at each income level we calculate the tax differential between employment and self-employment before and after reform, and use the extensive-margin parameter estimates to predict the share of individuals at each income level who would transition as a result of the reform.[23]

Importantly, for the calculation the extensive behavioural effects, we assume that on switching taxable income does not change. This seems supported by the Heckman-selection model of income presented in Appendix A, in which we do not observe large changes in income on transition from self-employment to employment. However, we treat it here mostly as an approximation. In a separate paper (Zawisza, 2017), where we conduct a more involved analysis of the general optimal tax system, we are explicit about how we model relationship between income levels in self-employment and employment for every individual.

As can be seen in Table 6, the mechanical effects of losses in revenue due to the tax cuts are 407.5 million zł for employment and 72.5 million zł for the self-employment tax base. In comparison, the reduction in revenue on the intensive margin are 14.6 million zł for employment and 7.3 million zł for self-employment. It is interesting to note that the contribution of the behavioural

[22] As has been already discussed in Section 3, the relevant formula can also be used to approximate the DWL resulting from these reforms, on the assumption that a local estimate of welfare effects is a reasonable approximation for the large tax changes which occurred in 2009.

[23] Specifically, we do this based on pre-reform numbers of taxpayers in each tax base, in 10,000zł bands.

Table 6: Calculations of DWL of reform.

Tax Base	Employment	Business
Mechanical effect		
1st tax band	-20,764,139	-6,755,519
2nd tax band	-286,883,745	-48,800,616
3rd tax band	-99,908,035	-16,905,894
Total	-407,555,918	-72,462,029
Behavioural effect		
Intensive Margin		
1st tax band	3,653,246	1,030,008
2nd tax band	29,700,905	14,927,247
3rd tax band	14,653,178	7,325,887
Total	48,007,329	23,283,143
Extensive Margin		
B to E	5,373,097	N/A
E to B	246,611	N/A
Total	5,619,708	

responses along the intensive margin for self-employment is magnified by the higher elasticities in this tax base, so that they account for almost exactly half of recouped revenue but less than a quarter of the lost revenue.

The table shows that on the extensive margin, switching serves to further offset the mechanical losses of tax revenue. In particular, switching from the business tax base to employment results in an increase in tax revenue of around 5.4 million zł. A smaller amount of around 0.25 million zł was also predicted to have stemmed from reduced switching from the employment to the business tax base. These quantities would imply that, in the immediate aftermath of the reform, 7.9% of the revenue gains were accounted for by the extensive-margin response. As has been already mentioned, if we were to see changes in utility due to behavioural responses as negligible for this reform, 7.9% is also the proportion of the reduction in DWL due to extensive margin behaviour.

8 Conclusion

This paper contributes to the literature on optimal taxation by addressing two outstanding questions in the literature on optimal taxation. First, it provides credible estimates of intensive-margin elasticities which allow us to control for income-growth in the income distribution which varies year-to-year. The methodology employed here allows us to examine heterogeneity in intensive-margin elasticities along two dimensions: between the employed and the self-employed, and for different income bands for the employed. Along the former dimension, elasticities for the self-employed appear to be roughly three times larger than elasticities for the employed. Along the latter dimension, there is surprising stability of the elasticity in the 0.21-0.25 region.

Second, it estimates the degree of switching in response to differential taxation between two important tax bases, that of employed taxpayers and the tax base of those who own a small business. The magnitude of the response is large, with a 10,000 zł ($ 2,670 as of May 2017) change in the relative tax burden implying a change in the probability of a taxpayer filing business income by 3.75 percentage points. This is a non-negligible response considering that the magnitude of taxpayers reporting self-employment income at median income levels is small (around 10%, which rises to around 50% in the top percentile of the income distribution). Finally, applying the theoretical sections of this paper, we explored how the intensive and extensive-margin estimates fit together in determining formulae for the revenue effects of the tax reform, as well as an approximation of the DWL. The higher elasticities for self-employment were found to make a disproportionately large contribution to the changes in tax revenue as a result of the 2009 Polish tax cuts, relative to employment.

A number of open questions remain which could have a bearing on the findings of this paper, and which would merit further exploration. An important feature of the analysis presented here is social security contributions were not treated as part of the tax system. However, while some types of contributions, such as those towards a defined-contribution pension scheme, in principle have value for individuals in terms of deferred future consumption, others, such as contributions towards disability insurance, are unlikely to be valued in the same way (and be more analogous to straightforward taxation). Since such social security contributions comprise a considerable proportion of a worker's salary, our estimates of intensive-margin elasticities for the employed would have to be adjusted. Secondly, the Polish tax system has an interesting feature in that social security contributions are fixed amounts for the self-employed. This is likely to create additional incentives to transition to self-employment which have not been modelled here (although the methodology for modelling the role of tax differentials would still apply, since it concerns absolute changes), even if they had been adequately controlled for in this study. Whether some of the changes in contributions levels in 2007-2008 may have also had an equivalent impact as the changes in tax rates would be an important question for further study.[24]

A further question is the role of risk for self-employed individuals, and the extent to which this interacts with features of the tax system. For instance, higher income volatility is a known feature of self-employment and, as a result, individuals are more likely to experience periods of low income or loss-making income when self-employed. Given this, there would plausibly be a role for the tax system in providing a degree of income insurance. In principle, this could be achieved at least in part by manipulating marginal tax rates. Whether or not this would provide additional rationale for preferential tax treatment of self-employment (in addition to greater income elasticities), and how this may be moderated by extensive-margin transitions, is a question plausibly addressed in a dynamic model. It is left for further work.

[24] A yet further question to answer is the extent to which progressivity, joint filing and the allowance of broader deductions are traded off *among the self-employed,* when they chose between linear and progressive tax schedules, and to what extent this impacts the amount of income they declare. This would help answer a further important question of how sensitive taxable income is to the breadth of the tax base, a question raised and examined in the context of US tax reforms in Kopczuk (2005).

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A Effect of Switching on Income

To identify the impact of moving from the business to the employment tax base on taxable income, the Heckman selection-correction approach presents a potentially suitable approach. In practice, the difference in income between employment and self-employment is only observed for those who switch tax bases. To the extent that switching is driven by unobservable factors affecting income growth, any estimate relying on those who do actually switch will be subject to selection bias (unobservables driving income growth may be correlated with unobservables driving switching). However, a Heckman selection-correction model with a suitable exclusion restriction may allow us to adjust for a term in the estimate of the income change on switching.

Here, we focus on the sample of individuals who were in self-employment in 2008 who (i) either filed under the linear schedule or (ii) filed under the progressive schedule, but filed with spouses and whose spousal income was low enough to guarantee that they were in the lower tax bracket in 2008. The former group is understood as the Treatment group, while the latter group is understood to be the Control group. This is analogous to the methodology for estimating the effect of the reform on switching from self-employment to employment presented above.

As in a classic Heckman model, the transition from the business to the employment tax base is modelled as the Probit model

$$Pr(trans_{it+s}^{B\to L}) = \Phi\left(\gamma + g(z_{it}) + \delta \cdot 1(i \in T)\right)$$

where the group T is defined as the individuals reporting income according to the linear tax schedule. The interaction term $1(i \in T) \cdot 1(t \geqslant 2009)$ will here be treated as an exclusion restriction. The intuition is that an individual owning a business under linear reporting is more likely to switch to employment than a business owner reporting under the progressive tax base with a low-income spouse. This is because they are more likely to fall into the higher tax brackets on switching than an individual sharing tax liability with a spouse, all other things being equal.

The model of the income process for the switchers is

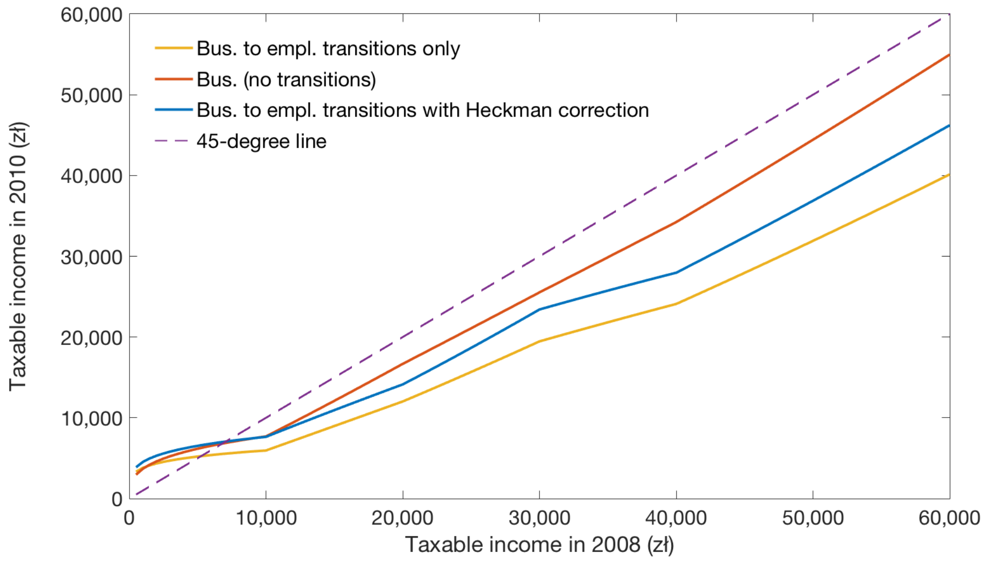
$$\Delta \log(z_{i,t+1}) = \alpha + \beta\lambda\left(\hat{\gamma} + \hat{g}(z_{it}) + \hat{\delta} \cdot 1(i \in T)\right) + f(z_{it}) + \xi_{i,t+1}$$

where $\lambda(.)$ is the estimated Mills ratio, $\lambda(.) = \phi(.)/\Phi(.)$ obtained from the estimated first-stage regression, $f(.)$ is a flexible spline in base-year income. The results to this estimation procedure are presented in Table 7, and the predicted levels of income on transition are shown in Figure 7.

B Social-Planner's Problem

The social planner's maximization problem is that of maximizing utility subject to truthful revelation by each type of individual, and subject to the

Figure 7: Income change on transition from business to employment.



resource constraint that all consumption is no greater than the output of the economy. This can be summarized as the following:

$$\max_{\{b(\theta),l(\theta),v_L(\theta),v_B(\theta),\tilde{\phi}_\theta\}} \int_\Theta \left[\tilde{G}(\tilde{\phi}_\theta)v^B(\theta) + (1-\tilde{G}(\tilde{\phi}_\theta))v^L(\theta)\right] d\tilde{F}\theta -$$

$$- \int_\Theta \left[\int_{\underline{\phi}}^{\tilde{\phi}_\theta} \phi \tilde{g}\left(\phi\right) d\phi\right] d\tilde{F}\theta \quad (9)$$

such that

$$v^{B'}(\theta) = \psi^B \left(\frac{e^B(\theta)}{\theta}\right) \frac{e^B(\theta)}{\theta^2} \qquad \forall \theta, \hat{\theta} \in \Theta \qquad (10)$$

$$v^{L'}(\theta) = \psi^L \left(\frac{l^L(\theta)}{\theta}\right) \frac{l^L(\theta)}{\theta^2} \qquad \forall \theta, \hat{\theta} \in \Theta \qquad (11)$$

$$\tilde{\phi}(\theta) = v^B(\theta) - v^L(\theta) \qquad \forall \theta, \hat{\theta} \in \Theta \qquad (12)$$

$$\int_{\Theta}\left[(G(\tilde{\phi}_{\theta})(\tilde{\omega}e(\theta)) + (1 - G(\tilde{\phi}_{\theta})l(\theta))\right] dF\theta -$$
$$- \int_{\Theta}\left[G(\tilde{\phi}_{\theta})c^{B}(\theta) + (1 - G(\tilde{\phi}_{\theta}))c^{L}(\theta)\right] dF\theta \geqslant 0 \quad (13)$$

with variables defined in the main text. Here, we have also introduced the notion of an 'original wage' in self-employment $e(\theta)$, defined as $e(\theta) = b(\theta)/\bar{\omega}$. This can be interpreted as the additional level of income generated by an extra hour of self-employment.

Truthful revelation is ensured by three incentive-compatibility constraints, two on the intensive-margin for each tax base, (10) and (11), and one on the extensive margin, (12). The first two are derived from the first-order conditions of individuals optimally choosing the amount of taxable income to declare in each tax base, while the latter is derived from the threshold condition at which an individual is indifferent between the two tax bases. The social planner's objective function involves purely the social welfare weights assigned to the indirect utilities, as well as to the fixed costs faced by individuals. It is important to note that in this derivation, we use the assumption that the level of taxable business income $b(\theta)$ is equal to the premium for self-employment multiplied by the 'original wage' in self-employment, $e(\theta)$. In other words, $b(\theta) = \tilde{\omega}e(\theta)$. This simplifies considerably the derivations.

Having applied integration by parts to (10) and (11), and incorporating the fact that $T^{L}(\theta) = v^{L}(\theta) + \psi^{L}\left(l(\theta)/\theta\right)$ and $T^{B}(\theta) = v^{B}(\theta) + \psi^{B}\left(e^{B}(\theta)/\theta\right)$, the resulting Lagrangian for the social planner's problem becomes

$$
\begin{aligned}
L = \int_{\Theta} \left[\tilde{G}(\tilde{\phi}_\theta)v^B(\theta) + (1-\tilde{G}(\tilde{\phi}_\theta))v^L(\theta)\right] d\tilde{F}\theta - \int_{\Theta}\left[\int_{\underline{\phi}}^{\tilde{\phi}_\theta} \phi\tilde{g}\left(\phi\right)d\phi\right] d\tilde{F}\theta - \\
- \int_{\Theta}\left[\mu^{L'}(\theta)v^L(\theta) + \mu^L(\theta)\psi^L\left(\frac{l^L(\theta)}{\theta}\right)\frac{l^L(\theta)}{\theta^2}\right] dF\theta - \\
- \int_{\Theta}\left[\mu^{B'}(\theta)v^B(\theta) + \mu^B(\theta)\psi^B\left(\frac{e^B(\theta)}{\theta}\right)\frac{e^B(\theta)}{\theta^2}\right] dF\theta - \\
+ \lambda^{RC}\int_{\Theta}\left[G(\tilde{\phi}_\theta)\left(\tilde{\omega}e(\theta) - v^B(\theta) - \psi^B\left(\frac{e^B(\theta)}{\theta}\right)\right)\right] dF\theta - \\
- \lambda^{RC}\int_{\Theta}\left[(1-G(\tilde{\phi}_\theta))\left(l(\theta) - v^L(\theta) - \psi^L\left(\frac{l^L(\theta)}{\theta}\right)\right)\right] dF\theta
\end{aligned}
$$

The social planner is assumed to be maximizing the Lagrangian by assigning levels of taxable income to individuals of each type, subject to truthful revelation and optimization on the extensive margin, while also assigning levels of indirect utility to each type. This procedure yields the same result as if the social planner simply set appropriate taxes and transfers, and allowed individuals to respond to them optimally on the intensive and extensive margins. Thus, the *control variables* of the problem are $\{b(\theta), l(\theta), v_L(\theta), v_B(\theta), \tilde{\phi}_\theta\}$. We derive the first-order conditions for each of these accordingly.

The first-order conditions for $v^B(\theta)$ and $v^L(\theta)$ are, respectively:

$$
\begin{aligned}
\mu^{B'}(\theta) = \tilde{G}(\tilde{\phi}_\theta)\tilde{f}(\theta) - \lambda^{RC}G(\tilde{\phi}_\theta)f(\theta) + \\
+ \lambda^{RC}g(\tilde{\phi}_\theta)f(\theta)\left[b(\theta) - c^B(\theta) - (l(\theta) - c^L(\theta))\right]
\end{aligned}
$$

$$
\begin{aligned}
\mu^{L'}(\theta) = \left(1-\tilde{G}(\tilde{\phi}_\theta)\right)\tilde{f}(\theta) - \lambda^{RC}\left(1-G(\tilde{\phi}_\theta)\right)f(\theta) - \\
- \lambda^{RC}g(\tilde{\phi}_\theta)f(\theta)\left[b(\theta) - c^B(\theta) - (l(\theta) - c^L(\theta))\right]
\end{aligned}
$$

We define the term $\Delta T(\theta) = T^B(\theta) - T^L(\theta) = b(\theta) - c^B(\theta) - (l(\theta) - c^L(\theta))$ as the tax differential between self-employment and employment. Integrating the above first-order conditions, the definition for $\Delta T(\theta)$ and the transversality conditions to maximization problem, $\mu^B(\underline{\theta}) = \mu^L(\underline{\theta}) = 0$ and $\mu^B(\bar{\theta}) = \mu^L(\bar{\theta}) = 0$, we obtain:

$$0 = \int_{\Theta} \left[\tilde{G}(\tilde{\phi}_\theta) f(\theta) - \lambda^{RC} G(\tilde{\phi}_\theta) f(\theta) + \lambda^{RC} g(\tilde{\phi}_\theta) f(\theta) \Delta T(\theta)\right] d\widetilde{F}\theta$$

$$0 = \int_{\Theta} \left[\left(1 - \tilde{G}(\tilde{\phi}_\theta)\right) f(\theta) - \lambda^{RC} \left(1 - G(\tilde{\phi}_\theta)\right) f(\theta) - \lambda^{RC} g(\tilde{\phi}_\theta) f(\theta) \Delta T(\theta)\right] d\widetilde{F}\theta$$

Adding the two conditions yields the result that $\lambda^{RC} = 1$. We incorporate this insight into the first-order conditions for $v^B(\theta)$ and $v^L(\theta)$. Integrating, we see that

$$\mu^B(\theta) = \int_{\Theta} \left[\tilde{G}(\tilde{\phi}_\theta) f(\theta) - G(\tilde{\phi}_\theta) f(\theta) + g(\tilde{\phi}_\theta) f(\theta) \Delta T(\theta)\right] d\widetilde{F}\theta$$

$$\mu^L(\theta) = \int_{\Theta} \left[\left(1 - \tilde{G}(\tilde{\phi}_\theta)\right) f(\theta) - \left(1 - G(\tilde{\phi}_\theta)\right) f(\theta) - g(\tilde{\phi}_\theta) f(\theta) \Delta T(\theta)\right] d\widetilde{F}\theta$$

Now, the first-order conditions for $e(\theta)$ and $l(\theta)$ are, respectively:

$$G(\tilde{\phi}_\theta) f(\theta) \left[\tilde{\omega} - \frac{1}{\theta} \psi^{B'} \left(\frac{e(\theta)}{\theta}\right)\right] = \frac{\mu^B(\theta)}{\theta} \left[\frac{1}{\theta} \psi^{B'} \left(\frac{e(\theta)}{\theta}\right) + \frac{e(\theta)}{\theta^2} \psi^{B''} \left(\frac{e(\theta)}{\theta}\right)\right]$$

$$\left(1 - G(\tilde{\phi}_\theta)\right) f(\theta) \left[1 - \frac{1}{\theta} \psi^{L'} \left(\frac{l(\theta)}{\theta}\right)\right] = \frac{\mu^L(\theta)}{\theta} \left[\frac{1}{\theta} \psi^{L'} \left(\frac{l(\theta)}{\theta}\right) + \frac{l(\theta)}{\theta^2} \psi^{L''} \left(\frac{l(\theta)}{\theta}\right)\right]$$

It is easy to rearrange this first-order conditions and introduce terms representing the elasticities of taxable income for the employment and self-employment tax bases. Dividing through by $\psi^{B'}(e(\theta)/\theta)/\theta$ and $\psi^{L'}(l(\theta)/\theta)/\theta$, respectively, we get:

$$\frac{\tilde{\omega} - \psi^{B'} \left(e(\theta)/\theta\right)/\theta}{\psi^{B'} \left(e(\theta)/\theta\right)/\theta} = \frac{\mu^B(\theta)}{\theta f(\theta) G(\tilde{\phi}_\theta)} \left(\frac{1 + \psi^{B''}(e(\theta)/\theta)(e(\theta)/\theta^2)}{\psi^{B'} \left(e(\theta)/\theta\right)/\theta}\right)$$

$$\frac{1 - \psi^{L'} \left(l(\theta)/\theta\right)/\theta}{\psi^{L'} \left(l(\theta)/\theta\right)/\theta} = \frac{\mu^L(\theta)}{\theta f(\theta) \left(1 - G(\tilde{\phi}_\theta)\right)} \left(\frac{1 + \psi^{L''}(l(\theta)/\theta)(l(\theta)/\theta^2)}{\psi^{L'} \left(l(\theta)/\theta\right)/\theta}\right)$$

Since the elasticities of self-employment income with respect to the net-of-tax rate, and the elasticity of employment income with respect to the net-of-tax rate, are:

$$\varepsilon^B(\theta) = \frac{\psi^{B'}\left(b(\theta)/\tilde{\omega}\theta\right)/(\tilde{\omega}\theta)}{\psi^{B''}\left(b(\theta)/\tilde{\omega}\theta\right)\left(b(\theta)/(\tilde{\omega}\theta)^2\right)}$$

$$\varepsilon^L(\theta) = \frac{\psi^{L'}\left(l(\theta)/\theta\right)/\theta}{\psi^{L''}\left(l(\theta)/\theta\right)\left(l(\theta)/\theta^2\right)}$$

respectively, we can immediately substitute them into the earlier expressions.

Finally, we may substitute into the first-order conditions for $e(\theta)$ and $l(\theta)$ the terms for the Lagrange multipliers on the incentive-compatibility constraints from the first-order conditions for the levels of indirect utility $v^B(\theta)$ and $v^L(\theta)$. This gives the two conditions presented as the solutions to the social planner's problem in the main text:

$$\begin{aligned}\frac{T'^B(\theta)}{1-T'^B(\theta)} = & \left[\frac{1+\frac{1}{\varepsilon_B}}{\theta f(\theta) G_\theta(\widetilde{\phi}_\theta)}\right]\times \\ & \times \int_{\underline{\theta}}^{\theta}\left[\left\{\widetilde{f}(\hat{\theta})\widetilde{G}(\widetilde{\phi}_{\hat{\theta}}) - f(\hat{\theta})G(\widetilde{\phi}_{\hat{\theta}})\right\} + f(\hat{\theta})\left(g(\widetilde{\phi}_{\hat{\theta}})\Delta T^{L,B}(\hat{\theta})\right)\right]d\hat{\theta}\end{aligned}$$

$$\begin{aligned}\frac{T'^L(\theta)}{1-T'^L(\theta)} = & \left[\frac{1+\frac{1}{\varepsilon_L}}{\theta f(\theta)(1-G_\theta(\widetilde{\phi}_\theta))}\right]\times \\ \times \int_{\underline{\theta}}^{\theta} & \left[\left\{\widetilde{f}(\hat{\theta})(1-\widetilde{G}(\widetilde{\phi}_{\hat{\theta}})) - f(\hat{\theta})(1-G(\widetilde{\phi}_{\hat{\theta}}))\right\} - f(\hat{\theta})\left(g(\widetilde{\phi}_{\hat{\theta}})\Delta T^{L,B}(\hat{\theta})\right)\right]d\hat{\theta}\end{aligned}$$

The limit conditions $T^B(\underline{\theta}) = T^L(\underline{\theta}) = 0$ and $T^B(\bar{\theta}) = T^L(\bar{\theta}) = 0$ are a consequence of evaluating at $\underline{\theta}$ and $\bar{\theta}$, and applying the transversality conditions $\mu^B(\underline{\theta}) = \mu^L(\underline{\theta}) = 0$ and $\mu^B(\bar{\theta}) = \mu^L(\bar{\theta}) = 0$.

Table 7: Heckman model of changes in income on transition between tax base.

	Heckman	OLS (non-switchers)	OLS (switchers)
Base-year (log) taxable income spline 1	0.202***	0.319***	0.195***
	(0.0653)	(0.0150)	(0.0295)
Base-year (log) taxable income spline 2	0.860***	1.115***	1.014***
	(0.122)	(0.0402)	(0.0875)
Base-year (log) taxable income spline 3	1.214***	1.050***	1.188***
	(0.247)	(0.0814)	(0.188)
Base-year (log) taxable income spline 4	0.586	1.020***	0.743***
	(0.408)	(0.131)	(0.279)
Base-year (log) taxable income spline 5	1.204***	1.168***	1.258***
	(0.188)	(0.0611)	(0.0900)
Base-year (log) business income spline 1	-0.00459	-0.0257***	-0.0763***
	(0.0983)	(0.00966)	(0.0140)
Base-year (log) business income spline 2	-0.381**	-0.186***	-0.468***
	(0.177)	(0.0391)	(0.101)
Base-year (log) business income spline 3	-0.363	-0.159*	-0.650**
	(0.390)	(0.0915)	(0.302)
Base-year (log) business income spline 4	-0.930	-0.390**	-0.323
	(0.780)	(0.166)	(0.459)
Base-year (log) business income spline 5	-0.777	-0.0333	-1.195***
	(1.529)	(0.273)	(0.116)
Mill's ratio	-0.318		
	(0.448)		
Constant	7.738***	6.305***	7.750***
	(0.596)	(0.124)	(0.247)
Observations	25,704	25,704	3,575
R-squared		0.314	0.376

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1